Math 103, Fall 2014
Week 9

In Class Work, Tuesday, October 21st

Exercise 1

To warm up, some derivatives:

(a) Find $\frac{d}{dx} \tan \sin(xe^x)$.

(b) Find $\frac{d}{dx} \frac{(\sin x)e^x}{(x^3+1)^2(x-2)}$.

(c) $\text{Ci}(x)$ is function with the property that $\text{Ci}'(x) = \frac{\cos x}{x}$. What is $\frac{d}{dx}\text{Ci}(x^2)$?

(d) What is $\frac{d}{dx}\text{Ci}(e^{x^2})$?

(e) What is $\frac{d}{dx}\text{Ci}(x \sin x^2)$?

(f) What is $\frac{d}{dx} xe^{-1/x^2}|_{x=0}$?

Exercise 2

tanh is a function you may not have encountered yet. $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$. Find $\frac{d}{dx} \tanh^{-1} x$.

(i) Find $\tanh' x$.

(ii) Using some algebra, manipulate your definition of $\tanh' x$ so that it is expressed in terms of $\tanh x$.

(iii) What is $\tanh' \tanh^{-1} x$? ($\tanh^{-1}$ is the inverse of $\tanh$. Remember to use the properties of inverses.)

(iv) Use the derivative rule for inverses to find $\frac{d}{dx} \tanh^{-1} x$?

Exercise 3

For each of the following piecewise functions, determine whether it is continuous at 1, and then draw a picture or plot on a calculator to confirm your answer.

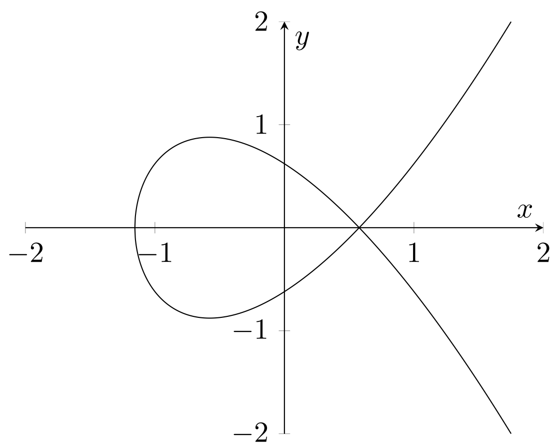
(a) $f(x) = \begin{cases} \sqrt{x} & \text{if } x < 1 \\ x/2 & \text{if } x \geq 1 \end{cases}$

(b) $g(x) = \begin{cases} e^x & \text{if } x < 1 \\ ex & \text{if } x \geq 1 \end{cases}$

(c) $h(x) = \begin{cases} x^2 & \text{if } x < 1 \\ x & \text{if } x \geq 1 \end{cases}$

Exercise 4

Consider this graph of points satisfying $y^2 = x^3 - x + \sqrt{\frac{4}{27}}$:



(a) Before taking the derivative, use the graph to identify where you think the derivative of this curve will be defined.

(b) Use implicit differentiation to find the derivative $\frac{dy}{dx}$? (Your final answer should be $\frac{dy}{dx} = \cdots$ with only x and y on the right.)

(c) Compare the points where $\frac{dy}{dx}$ isn't defined to the points on the curve where you didn't expect the derivative to be defined.

(d) Use implicit differentiation to find the derivative $\frac{d^2y}{dx^2}$. (Your final answer should be $\frac{d^2y}{dx^2} = \cdots$ with only x and y on the right.)

Exercise 5

In a simple economic model, there is a relationship between the *price* p of widgets and the *quantity* $Q(p)$ of widgets that will be sold at price $Q(p)$.

(a) What are the units of $Q'(p)$? For most goods, is $Q'(p)$ positive or negative?

(b) The *elasticity of demand* is the value $E(p) = \frac{p}{Q(p)}Q'(p)$. This value measures change relative to the original values. For example, if $E(p)$ is always equal to -1, that means that when the price increases by 1%, the quantity will decrease by 1%. What are the units of elasticity?

(c) Suppose $Q(p) = 3p^{-1/2}$. Find $(\ln Q(p))'$.

(d) Remember that $(\ln Q(p))' = \frac{Q'(p)}{Q(p)}$. What is $E(p)$?

Exercise 6

If you throw a ball at velocity v at an angle θ, the object will land $\frac{v^2 \sin 2\theta}{g}$ meters away (where g is the gravitational constant). We are aiming at a target $\frac{50^2}{g}$ meters away, so if we throw the ball at the angle $\pi/4$ with $v = 50$, we will exactly hit the target. Which throw will get us closer: if we get the velocity exactly right but the angle wrong by 0.2 meters or if we get the angle exactly right but the velocity wrong by 0.2 radians? (Note that you *don't* need a calculator for this problem.)