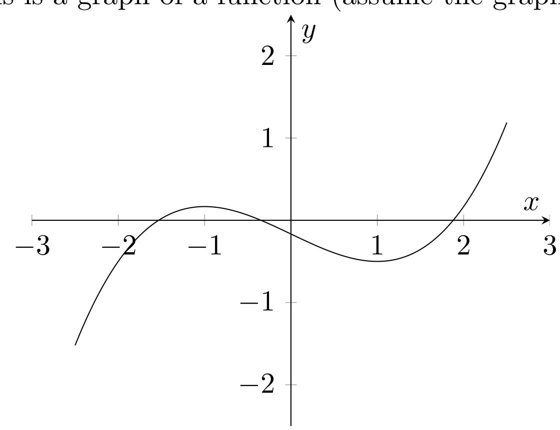
Math 103, Fall 2014
Week 9

In Class Work, Thursday, October 23rd

Warm Up

This is a graph of a function (assume the graph continues off the page:



(a) What are the local minima on $(-\infty, \infty)$?

(b) What are the absolute minima on $(-\infty, \infty)$?

(c) What are the local minima on $(-\infty, 0]$?

(d) What are the absolute minima on $(-\infty, 0]$?

(e) What are the local minima on $[0, \infty)$?

(f) What are the absolute minima on $[0, \infty)$?

Exercise 1

Consider the function $g(x) = \frac{x}{x^2+2}$.

(a) Identify the absolute minima and maxima of $g(x)$ on $[-10, 10]$.

  (i) `First, find` $g'(x)$`.`

  (ii) `Find the points where` $g'(x) = 0$`.`

  (iii) `Find the points where` $g'(x)$ `is undefined.`

  (iv) `Make a table of the values of` $g(x)$ `at the endpoints` -10 `and` 10`, and at each critical points which is in the interval` $(-10, 10)$`.`

  (v) `The absolute minimum is the smallest value you find in that table.  The absolute maximum is the largest value.`

(b) Identify the absolute minima and maxima of $g(x)$ on $[-1, 1]$.

Exercise 2

Sketch a graph $u(x)$ with domain $(-\infty, \infty)$ such that all of the following happen:

- $u(x)$ has local maxima at $x = 0$ and $x = 4$,
- $u(x)$ has a local minimum at $x = 2$,
- $u(x)$ has an absolute maximum at $x = 0$,
- $u(x)$ has no absolute minimum.

Exercise 3

The exact wording of the Extreme Value Theorem is:

> If $f(x)$ is a continuous function on a closed interval $[a, b]$, then f attains both an absolute maximum value M and an absolute minimum value m in $[a, b]$.

This theorem has two important assumptions: that f is continuous and that the interval is closed.

(a) Sketch the graph of a function which is defined on $[-2, 2]$, is *not continuous* but *has both* an absolute maximum value and an absolute minimum value on $[-2, 2]$.

(b) Sketch the graph of a function which is defined on $[-2, 2]$, is *not continuous* but *has neither* an absolute maximum value nor an absolute minimum value on $[-2, 2]$.

(c) Suppose you have a function $f(x)$ which is not continuous on $[a, b]$. What do you know about the existence of absolute minima and maxima on on $[a, b]$? (For example: do you know that f does have an absolute maximum? Do you know that f does not?)

(d) Sketch the graph of a function which is defined on $(-2, 2)$, is *continuous* and *has both* an absolute maximum value and an absolute minimum value on $(-2, 2)$.

(e) Sketch the graph of a function which is defined on $(-2, 2)$, is *continuous* but *has neither* an absolute maximum value nor an absolute minimum value on $(-2, 2)$.

(f) Suppose you have a function $f(x)$ which is continuous on (a, b). What do you know about the existence of absolute minima and maxima on on (a, b)? (For example: do you know that f does have an absolute maximum? Do you know that f does not?)

(g) Consider the following argument:

> The Extreme Value Theorem is wrong for a constant function like $f(x) = 4$, because a constant function has no minimum or maximum value on $[-2, 3]$.

Explain (in sentences, in a way another student in the class could understand) what is wrong with this argument.

Put up a green flag before going on.

Exercise 4

Consider the function $h(x) = x^{4/3} - x^{2/3}$.

(a) Identify the absolute minima and maxima of $h(x)$ on $[-8, 8]$.

(b) Identify the absolute minima and maxima of $h(x)$ on $[-1, 1]$.

Exercise 5

Bert has a 144-inch length of flexible wire that he will use to frame a cylindrical laundry bin with to top or bottom. First he will bend the wire to make a rectangular frame, across which he will stretch a piece of cloth. Then he will roll the rectangle into a cylinder by bringing a pair of opposite sides together. Can Bert maximize the volume of the laundry bin? If so, what dimensions should the rectangle have in order to do so?

(i) Call the length of the side that gets rolled into a cylinder x and the length of the other side y. Write an equation for the radius r of the cylinder in terms of x.

(ii) Write an equation for the height h of the cylinder in terms of y.

(iii) Give an equation for the voume of the cylinder in terms of the dimensions x and y of the rectangle. (Hint: The volume of a cylinder with base r and height h is $\pi r^2 h$.)

(iv) Since $2x + 2y = 144$, $y = 72 - x$. Substitute this in to your previous equation to get the volume $V(x)$ as a function of x.

(v) Find $\frac{dV}{dx}$.

(vi) Find the critical points of $V(x)$.

(vii) Use this information to answer the question.