Math 110, Fall 2014
HWK01 due WED 28 January

On this homework set, problems 1, 3 and 5 involve computation, while problems 2, 4 and 6 involve mostly concept explanation. The point is that these two are equallly important!

1. (a) What approximate value do you get for $\sqrt{105}$ if you use a linear approximation to the function $\sqrt{x}$ near $x = 100$? Now use a calculator to find a more precise value and state the absolute error of the approximation and the relative error (error as a proportion lf the correct value).

   (b) In the same manner, estimate $1/105$. Say what approximation you are using, centered where, and give the absolute and relative error.

2. Suppose I am trying to buy up land near the airport for a commercial development. Let $f(x)$ denote the amount of land (in acres) I can buy for x million dollars.

   (a) Would you expect the graph of f to be concave upward, downward or flat? Explain. Almost any answer can be right for this problem if accompanied by a plausible explanation.

   (b) Suppose $f(1) = 18$ and $f(0.9) = 17$. What would you estimate $f'(1)$ to be? Use this to estimate $f(2)$. State whether this is likely to be an over-estimate or an under-estimate of the actual value, and justify this using mathematics and your previous answers.

   (c) Write a formula for the marginal cost per acre after you own y acres of land (the formula should involve the function f, its derivatives, etc.).

3. Compute the following limits. Please indicate the steps of your derivation. The last one is tricky because you have to multiply and divide by $\sqrt{x^2+x}+x$ (who'd have thought?) and you're still not out of the woods (you're about a sandwich theorem away).

(a) $\lim_{x\to 3} \dfrac{x^2-9}{x^2-4x+3}$

(b) $\lim_{x\to 5^-} \dfrac{|2x-10|}{x-5}$

(c) $\lim_{x\to\infty} \dfrac{2x^2+3x+4}{5x^2+6x+7}$

(d) $\lim_{x\to\infty} \ln(3x)-\ln(x)$

(e) $\lim_{x\to\infty} \sqrt{x^2+x}-x$

4. Which of these functions would you expect to have a limit as $t \to \infty$ and why? Again, more than one view can be considered correct if the justification is plausible and matches the answer.

   (a) $f(t) =$ the speed at time t of a rocket, initially fired fast enough to escape the gravitational field of the earth (and the sun).

   (b) $f(t) =$ the amount of water in an infinite tank that is being filled at a constant rate and leaks faster the more water there is in it.

   (c) $f(t) =$ the temperature of a bowl of warm Jell-O left to sit on a kitchen counter.

   (d) $f(t) =$ the position of the tip of a spring after you give it a tweak and it starts vibrating back and forth.

5. Use L'Hôpital's rule, perhaps more than once, to evaluate these limits.

(a) $\lim_{x \to 1^+} \frac{\ln x}{\sqrt{x-1}}$

(b) $\lim_{x \to \infty} \frac{x^3}{e^x}$

6. Read the problem slowly and try to solve it. After 10 minutes, if you haven't solved it, go to the next page where the problem is broken down for you and solve it using the steps given. Understanding what constitutes a correct answer is really more of an issue of logic and notation than of computation.

> Let $f(A)$ be the time it takes for a machine to harvest an area A in a field of soybeans. Let $g(A)$ be the time it takes a single worker to do this by hand. Do not assume f and g are linear, as there may be influences from learning, fatigue, and so forth. Do assume that workers and machines can join forces by working independently on different parts of the field. Write a formula for the amount of time it takes a machine plus five workers to harvest an area, A. The unspecified functions f and g will of course appear in the formula, as may one or more inverse functions.

Let's say we're measuring area in acres and time in hours.

(a) How many acres can a worker harvest in an amount of time, t?

(b) How many acres can a machine harvest in an amount of time, t?

(c) How many acres can five workers and a machine harvest in an amount of time, t? This is where the part of the problem that begins "Do assume..." is important.

(d) Give the function in the previous part a name: let $L(t)$ be the number of acres that a machine and five workers can harvest in t hours. What function tells how many hours it takes for a machine and five workers to harvest A acres?

(e) Write the answer to the question that was asked in a sentence: the number of hours it takes for a machine and five workers to harvest A acres is ... where $L(t)$ is the function ....