# Math 110, Spring 2015
# HWK06 due WED 04 March

1. Compute the Taylor polynomials as indicated.

   (a) $f(x) = \tan x$, $n = 3$, $a = 0$

   (b) $f(x) = \tan x$, $n = 2$, $a = \pi/4$

   (c) $f(x) = \ln \cos x$, $n = 3$, $a = 0$

   (d) $f(x) = (x + x^2)^{2/3}$, $n = 2$, $a = 1$

2. Compute these Taylor polynomials by making use of substitution, addition and/or multiplication.

   (a) $f(x) = \cos(x^3)$, $n = 8$, $a = 0$

   (b) $f(x) = (1 - 3x + 2x^2)\sin(x)$, $n = 3$, $a = 0$

   (c) $f(x) = (e^x - 1 - x)^3$, $n = 7$, $a = 0$

3. Use Taylor polynomials to approximate the following values without using a calculator. In each case, indicate your choice of $f, n$ and $a$, choosing $n$ large enough that you expect to be within 2% of the true value (you don't need a rigorous bound - it's OK if you're in the ballpark).

   (a) $\sqrt[3]{10}$

   (b) $\arctan(0.95)$

   (c) $\log_2 65$

   (d) $\left(\frac{17}{20}\right)^{2.1}$

4. A company uses a continuous time model to forecast its revenue, where $f(t)$ is the instantatnous rate of revenue in mega-dollars per year. Based on their trend data over the last year, they are able to estimate the present revenue rate and its first few derivatives:

$$\begin{aligned} f(0) &\approx 45 \\ f'(0) &\approx 15 \\ f''(0) &\approx 6 \\ f'''(0) &\approx 3 \end{aligned}$$

In order to assess a buyout offer, the company would like to forecast the revenue they will receive over the next year (NOT the instantaneous rate after one year but the total over the coming year). Please provide an estimate of this, clearly stating your methods and assumptions.

5. Let $\Phi(x) = \int_{-\infty}^{x} \sqrt{\frac{1}{2\pi}} e^{-t^2/2}\, dt$ denote the probability that a standard normal random variable is at most $x$.

(a) Compute the cubic MacLaurin polynomial $P_3(x)$.

(b) Using a calculator, compute the values of $P_1(1)$ and $P_3(1)$.

(c) Give bounds on the actual value of $\Phi(1)$ by using Taylor's theorem with remainder, with $n = 3$. You may use a calculator.

6. Complete the work we began in class by answering the following questions. Let $f(x) = \cos x$ and let $a = \pi/4$.

   (a) Write $P_1(x)$ in as simple a form as possible, but exact (do not use decimal approximation).

   (b) Plug in $x = \pi/5$ and simplify again, to get an exact value for $P_1(\pi/5)$.

   (c) Use Taylor's Remainder Theorem to write an exact expression for $R_1 = f(\pi/5) - P_1(\pi/5)$ involving an unknown value, $u$, between $\pi/4$ and $\pi/5$.

   (d) Write down upper and lower bounds on $R_1$.

   (e) Plug in a numerical value for $P_1(\pi/5)$ and the upper and lower bounds on $R_1$ to get upper and lower bounds on $\cos(\pi/5)$.