

**Math 110, Fall 2014**  
**Homework for MON 01 December**

1. Let  $f(x, y) = \sqrt{x^2 + xy + y^2}$ .

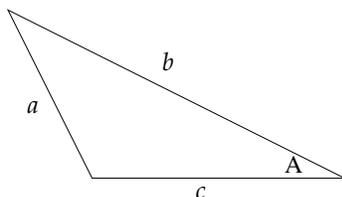
(a) Compute  $\frac{\partial f}{\partial x}$  and  $\frac{\partial f}{\partial y}$  at the point  $(3, 5)$ .

(b) Use the partial derivatives to estimate  $f(3.1, 5)$ .

(c) Use the partial derivatives to estimate  $f(2.9, 5.2)$ .

(d) Use the partial derivatives to evaluate the slope of the tangent line to the curve  $f(x, y) = 7$  at the point  $(3, 5)$ . This will be an exact computation, not an estimate.

2. Finish problem #67 from Section 14.3 of the book:

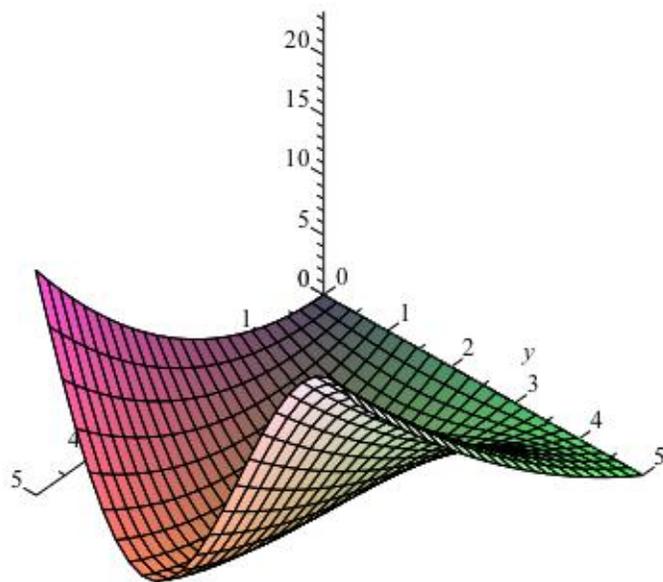


Let  $a, b$  and  $c$  be the three sides of a triangle and let  $A$  be the angle opposite  $a$ .

- (a) Write an equation relating these four quantities via the law of cosines. Move all the terms to one side so it says that something is equal to zero.
- (b) Suppose  $a, b$  and  $c$  are free to vary, so that  $A$  is a function of these three quantities. Take the partial derivative with respect to  $A$  of the equation you wrote in the previous step. It should say that something new is equal to zero.
- (c) Compute  $\partial A/\partial a$  by solving this equation for  $\partial A/\partial a$ .
- (d) Compute  $\partial A/\partial b$  in a similar manner.

3. The picture shows the graph of  $z = f(x, y)$  above the square  $0 \leq x, y \leq 5$ . Use it to estimate (eyeball) the following partial derivatives. Note: the  $x$ -axis is the one mostly hidden behind the graph.

(i)  $\frac{\partial z}{\partial y}(0, 3)$ ; (ii)  $\frac{\partial z}{\partial y}(5, 0)$ ; (iii)  $\frac{\partial z}{\partial x}(1, 0)$ .



4. The number of visitors to a national park depends on the ticket prices and the level of staffing. Let  $V, T$  and  $S$  denote these variables (in respective units of people per year, dollars and full time employees).

(a) What is the interpretation of the quantity  $\frac{\partial V}{\partial T}$ ?

(b) Is  $\frac{\partial V}{\partial T}$  likely to be greater or less than zero?

(c) What is the interpretation of the quantity  $\frac{\partial V}{\partial S}$ ?

(d) Is  $\frac{\partial V}{\partial S}$  likely to be greater or less than zero?

(e) In an effort to make the park self-supporting, Congress has pegged the staffing to ticket prices via the formula  $S = 10 + 3T$ , where  $S_0$  and  $k$  are constants. Explain, in this context, the meaning of the total derivative  $\frac{dV}{dT}$  and give a formula for this in terms of the partial derivatives of  $V$  with respect to  $T$  and  $S$ .

(f) Suppose  $V(T, S) = 50,000 e^{-T/10} \frac{S}{S + 45}$ . Compute  $\frac{dV}{dT}$  when  $T = 15$

5. Suppose that  $x$  and  $y$  vary along a piece of the curve  $x^3 - 3xy + y^3 = 3$  in the first quadrant.

(a) Differentiate implicitly using  $x$  as the independent variable and  $y$  as the dependent variable.

(b) Use this to evaluate  $dy/dx$  at the point  $(1, 2)$ .

(c) Now differentiate implicitly with  $y$  being the independent variable and  $x$  the dependent variable and use this to evaluate  $dx/dy$  at the point  $(1, 2)$ .

(d) Explain the relation between the answers in parts (b) and (c).

6. (Problem 66 from Section 14.4 of the textbook): Find the value of  $\partial x/\partial z$  at the point  $(1, -1, -3)$  if the equation

$$xz + y \ln x - x^2 + 4 = 0$$

defines  $x$  as a function of the two independent variables  $y$  and  $z$ .

7. In each case, draw the branch diagram, then evaluate the total or partial derivative requested.

(a)  $f(x, y) = x^3 + 2y^3$  with  $x = \cos(u)$  and  $y = \sin(u)$ . Compute  $\frac{df}{du}$ .

(b)  $f(x, y, z) = \frac{x}{\sqrt{x^2 + y^2 + z^2}}$  with  $x = \sin \theta \cos \phi$ ,  $y = \sin \theta \sin \phi$ ,  $z = \cos \theta$ .

Compute  $\frac{\partial f}{\partial \theta}$  and  $\frac{\partial f}{\partial \phi}$  at the point  $(\phi, \theta) = (0, \pi/6)$ .

8. Suppose the sports-metric utility function for a point guard is  $a^2p$  where  $a$  is the number of assists per game and  $p$  is the number of points per game. Michael Carter Williams averaged 7.5 assists and 15 points per game for the 76ers before being traded. What is his marginal rate of substitution of points versus assists if he is to maintain the same value according to this metric?