

Math 110, Spring 2015
HWK12 due WED 29 April

1. Let \mathbf{v} be the vector $14\hat{\mathbf{i}} - 3\hat{\mathbf{j}}$, let \mathbf{w} be the vector of length $5\sqrt{2}$ in the “Northeast” diagonal direction, and let u be the unit vector pointing in a direction fifteen degrees below the $\hat{\mathbf{i}}$ direction.

(a) Draw these three vectors.

(b) Compute $\mathbf{w} \cdot \mathbf{u}$.

(c) Compute $\mathbf{v} \cdot \mathbf{w}$.

3. Find the maximum of the objective function $x + 3y$ on these curves.

(a) $x(t) = 1 - t^2$, $y(t) = t^3$, $0 \leq t \leq 1$.

(b) $y = 4 - x^2$, all real x .

(c) $x^2 + xy + y^2 = 12$.

4. Suppose that a new graduate's job satisfaction is modeled as a function $u(x, y)$ where x is the annual salary and y is the amount of hours worked per year. Our model assumes ¹ that

$$u(x, y) = (4200 - y) * x^{0.6}.$$

- (a) Compute ∇u .
- (b) Evaluate $\nabla u(x, y)$ at the point $x = \$100,000$ and $y = 4000$. Please simplify fractional powers when there is a nice simplification.
- (c) The indifference curve through this point is the set of (x, y) having the same utility function. What is the slope of the indifference curve at the point $(100000, 4000)$ and what does this say about how much you'd have to increase the salary to get this overworked Wharton graduate (4000 hours is an 80 hour work week) to work each extra hour?

¹The power 0.6 is due to empirical data on the marginal value of a dollar, and the factor of $4000 - y$ is due to the assumption that the most a person could reasonably work on a regular basis is 84 hours (12 hours/day, 7 days/week), therefore $4200 - y$ represents the amount of free time.

5. What is the maximum value of $f(x, y) = xe^{-x^2-y^2}$ on the unit circle?

6. Suppose x and y are respectively the number of vacation days per year I spend in Western Pennsylvania and the number of vacation days I spend at Club Med. Due to cost constraints, $x + 5y$ cannot exceed 22. These are averages so the numbers need not be integers. If my utility is xy^2 then how many days should I spend at each?