Tips for Active Learning Classes at the College Level

1 Basics

1.1 The philosophy

It used to be that introductory math teaching at most US colleges was accomplished strictly by lecturing. In the last few years, there has been a movement toward *active learning*. This means different things at different places, from flipped classrooms to peer instruction, hands-on activities, problem-based curricula, and inquiry or discovery based teaching. At the K-12 level, these are all common, and in fact no one would think of teaching math to elementary school students purely by lecturing.

Lecturing works well when students have reached a certain level of mathematical maturity. At the graduate and advanced undergraduate levels, active learning is inefficient. At the freshman and sophomore levels the issue is more in flux. Math majors and STEM students may already interact with new mathematics by thinking through concepts, trying examples and asking questions. Many non-STEM students are not at this level.

Our philosophy is to continue to use active learning techniques for these students until they are at the point where they can take over these higher functions for themselves. In fact, the promotion of these attitudes and meta-skills is a main goal of freshman mathematics classes, along with the specific curricular goals. We want students to be able to reason and prove, to translate between words and symbols, to perform algebraic manipulations with correct justifications in mind, and to solve problems that are not clones of problems they have been shown how to do.

These notes promote one particular format of active learning class. It is a problem-based curriculum where students work in small groups with one or more instructor circulating among the groups. This is the main mode of our classes, but we also spend time lecturing and in whole-class discussions.

1.2 Disclaimer

In training one's self or others to teach active learning classes, it seems no amount of preparation or advice can substitute for a certain kind of on-the-job training. The essential ingredients of on-the-job

training are criticism and imitation. Experienced instructors visit the classrooms of new instructors, taking extensive notes on what they see: what went right, what went wrong, what might have worked better, and so on. New instructors also visit the classrooms of experienced instructors, taking equally careful notes on what went well or poorly, what might have worked better, and on what techniques they see used that they would like to use themselves. Some of this type of work can be done beforehand, via visits in the previous semester or videotaped classes, but it tends to be more valuable when it comes after the new instructor has had a chance to try teaching a class or two first.

If you are not going to adhere to this kind of a plan, then these notes will be of minimal use. You can't learn to play the piano by reading books about it or by talking about technique with Rubenstein, so don't try to learn a significant new teaching skill without practice and guidance either. This should not come as a surprise: it is essentially the same reason we use active learning in the first place. On the plus side, all the instructors here who have taught active learning classes (admittedly a self-selected sample) have become pretty good at it without too much overhead in terms of extra time per course. You can be pretty confident that you'll be able to step right in and teach both efficiently and effectively, as long as you use the peer support outlined above.

1.3 Typical classroom mechanics

A usual 50 minute class looks something like this. The first few minutes might consist of the instructor explaining some things that the students will need in order to do the worksheets for that day. Some variations are possible. If it is a flipped class (students make a first pass at the material at home) there might be some Q & A. A more inquiry based class might set the students to work right away and break for questions later.

Students are then given a problem or a worksheet (a sequence of problems) on which they work in groups of three or four students. The instructor circulates among the groups. This involves answering questions when necessary, doling out encouragement when necessary, challenging the students to justify what they claim to have figured out or to explain their half-baked ideas. Often the mere presence of the instructor encourages a renewed attack on a problem.

Depending on how things go, this might continue for the entire period. On other occasions there may be a need for some discussion among the whole class at various junctures because conceptual points arise that are better handled in a large group discussion. Another reason for pulling together for a

whole class discussion is if the worksheet contains a problem of the type that different groups solve in very different ways. Student presentation of solutions has a lot of advantages in this situation.

Often there are parts of the worksheet that are not covered in class. Worksheets are designed to contain enough material to extend some of the ideas, if time permits, for groups that finish faster than others. This flexibility is necessary so that class time is not wasted, because Some of the items not covered may be assigned for homework; others will be discarded. It is a good idea to make sure the students are aware of this in advance, so they don't think the instructor is being arbitrary or is taking liberties with the curriculum.

Some instructors prefer to go back and forth a little more, starting in a small group, then convening the large group when most small groups are done with the first problem, discussing it a bit, then remanding the class into small groups, and so on. When a class meets for 75 minutes twice a week instead of 50 minutes three times a week, it is usually necessary to go back and forth this way, and it is also often convenient to continue a large group discussion from the end of one class at the beginning of the next.

Whole class discussions come in different varieties. A more Socratic style discussion usually begins with the students explaining what they have done. Other students are required to listen carefully and to register agreement, disagreement or lack of comprehension. Once the explanation is comprehensible, those in disagreement are encouraged to justify their disagreement, with the aim of a resolution or synthesis. The instructor plays moderator as long as fruitful ideas are being produced, but slips into the role of leader when needed. For example, if no one challenges something wrong, or if there is a disagreement but it is too inarticulate to produce a good synthesis, then the instructor may rephrase what has been said so as to sharpen the contradiction or caricature a wrong approach, in a way that forces a light to dawn for at least some students. Whole class discussion can also involve a back and forth with the instructor providing the questions and students answering. Because the ability to formulate questions is a course goal, the instructor should not always provide the questions; however, good questioning is taught partly by modeling so this is an important mode as well. Lastly, there is a lecture mode, most useful when there is a need to wrap up a problem in a way that provides closure.

I will not say much about scheduled recitation sections because the course structure varies a lot from place to place. At Penn, there will be one or two 50-minute recitations per week, handled by the TA. These make life particularly easy for active learning instructors! The TA (who also comes to class and helps to circulate) knows what the students didn't understand during class and can expand on that during recitation. Also, when time runs out before a part of the worksheet that is not optional,

the TA can be asked to handle it in recitation. If your course is not structured with recitations, you may want to do so internally, designating one block of class time per week as recitation, and treating it accordingly.

1.4 Highly recommended procedures

Before getting into specifics of classroom technique in the next section, here are a few simple procedures that make a large difference.

- *Nametags.* Have the students wear nametags each day until you know their names (in my case four or five weeks). When calling on students, call on them by name and in general attempt to use their names frequently. This serves two purposes. First, by attempting to learn their names, you create a separate mental category for each student, which helps you pay attention to how each student is doing and to their individual needs. Secondly, there seems to be a psychological advantage to students hearing their names. Coming around to a small group and asking "Amy, can you tell me what progress your group has made?" or asking in a large group discussion whether "Sam's objection to Cindy's idea" holds water elicits more of a response than the same questions without the names identified. Somehow, students are more prone to take their own beliefs seriously when names are attached.

- *Randomize groups.* I usually assign groups using playing cards[1] randomly dealt: all the aces for a group, the twos form another, and so on. I re-form groups twice or thrice during the semester. Preventing students from choosing their buddies for a group helps them form connections, subject their ideas to the intellectual marketplace, and treat all the others in their groups fairly. The playing cards themselves lend an air of intrigue as students await the results of the lottery.

- *Get enough sleep.* Alertness is required on the part of the instructor. You'd be surprised what a difference this makes. You can fake it when you're lecturing, but try playing moderator when you can't concentrate or respond quickly, and you'll see what I mean.

- *Start the semester with a bang.* That is, don't spend the first day on administrative stuff and the second on some kind of review. Jump into an absorbing problem on Day 1, preferably a

[1]The playing cards are a holdover from a previous era when I had a little extra time. Nowadays I randomize via spreadsheet so that I have an automatic and resortable record of who is in what group. I am still tempted to go back to playing cards though.

tried-and-true chestnut, and fill in the administrative details later, when they've gotten the idea of what the class will be about. The tone of the first day's discussion sets an example that's hard to erase, so make sure it is as lively as possible. One way this can work out easily is if the schedule has the recitation meeting before the first class. Then you can cannibalize the recitation for the "nuts and bolts" session and start fast in the first lecture session.

- *Be yourself* at least up to a point. If you're the goofy type, be goofy, if you're serious and intense, let them feel the intensity, if you're understated and direct, be that way. You are on stage, and want to use your charisma, but don't pretend to be someone you're not.

2 Class composition and small group dynamics

2.1 Doing the rounds

When you first assign a problem to work on in small groups, there may not be much you have to do. No one is stuck yet; no one needs your help. There is a lot you can accomplish in this time. First, you can quickly take attendance group by group - after you know the names this takes less than half a minute[2]. After this, you will want to quickly "do the rounds". Visit each group once just to check that they have gotten down to work. On the first round, look for any trouble with the wording of the problem that may be holding people up. If it's part of their job to decipher it, encourage them to do so. If it's a mistake, or if you need to supply a definition, then make a quick announcement. On your second round you can linger longer. This is a good time to make a mental note of which groups are going faster than the others. It helps, during the subsequent large group discussion, to have a good idea of who has gotten how far. If a group has quickly and incorrectly or incompletely answered a problem and gone on to another, this is a good time to ask (innocently) for one of them to summarize for you what they found. The correct question on your part can cause them to re-examine what they've done without feeling that you've invalidated their answer. [Point of philosophy: you want them to be able to criticize their own work, realizing that mathematics will determine whether they are right, and that what they discover about this cannot be overruled by the teacher.]

Example: A group has quickly concluded that there no solutions, but they have used the incorrect

[2]Better yet, this is a job your TA can do while you're in the opening phase, even before the groups get to work. Taking attendance early has the added advantage of penalizing lateness.

manipulation

$$(a+b)^2 = a^2 + b^2.$$

This might be a good time to hand them a solution and have them figure out how to reslove the contradiction. Don't settle for "Gee, Teacher, your solution works so I guess we did something wrong." Instead, insist that it is you or the laws of algebra at fault. When they are able to pinpoint what went wrong, ask them to prove to you that $(a+b)^2 = a^2 + b^2$ really is not a law of algebra.

If there is only one solution and you don't want to spoil it for them, you may be able to hand them only the part of the solution that contradicts their error. If they are correct that there is no solution but haven't adequately argued it, tell them that there is indeed a solution, namely $a = 51$. When that doesn't work, try 52. Ask them after that whether they plan to do this for all natural numbers or whether they can give you a reason why none will work.

2.2 "Help, we're stuck"

The twin dangers here are that lazy students will say they're stuck so you do the work for them, while students who are truly stuck will lose morale if they have to sit idly during class. Asking if they have any ideas on what they might try will prove embarrassing if the answer is no. Sometimes I do this anyway. Sometimes I replace the problem with a smaller one: if you knew that $A = 15$ could you do the problem? Can you do the problem if you aren't required to make the number of cows and chickens the same? Sometimes I guess why they're stuck: So the problem is you don't really know the definition of average speed? An example of what might happen here is that they did know this but didn't think of going back to definitions as a way to proceed. Now when they say no that's not the problem, we know the definition of average speed, I say, "Oh, then you must be saying you don't have any way of determining the quantities defining speed, such as the time or the distance." They then say how they will proceed on this and I can smile and leave. In other cases, they are stuck because they don't really understand what's being asked. You can ask them to rephrase it, or ask them how they would check if someone else's answer (here you specify it) was right. It helps to have snooped enough so you have a good guess of where they're stuck. If not, you can ask them but won't always get reliable information.

If you have previously addressed problem-solving tactics in this class, then you might suggest to the students that they get out their list of problem-solving tactics and try some. It will do the students more good if you ask them to try a technique than if you tell them more precisely what to do, so try to keep your list of techniques down to a few that almost always help. Doing a problem with

numbers instead of variables, doing a problem with smaller numbers instead of the given numbers, or sheer trial and error, are almost always useful. It can also help to give them part of the answer. Maybe you can tell from their crossed out work that they are reluctant to integrate by parts because they see it will lead to something complicated. That's a good time to say, "By the way, it wasn't such a bad idea to integrate by parts, you know the integral of $1/(1+x^2)$ is in the table in the back of the book."

2.3 Getting groups to work together

Some populations tend to work well together. Others are less accustomed to this, less inclined to it, or simply don't see the point. One thing that makes a big difference is some serious attention to this during the "nuts and bolts" session at the beginning of the semester. This should include not only the syllabus, practices and grading procedrues, but also the motivation for the class and why you're doing it this way. I have a pretty elaborate set of slides and accompanying patter for Math 110 (business calculus), a link for which may be found near the link to thes notes. It includes a motivational skit that I perform with my TA's, as well as an explanation of the course goals and how the active learning format promotes these.

When you come across a particular small group that isn't working well together, here are some things to try. First, change the geometry so that the students are huddled together. If they have separate desks, pull them in a tight circle. If they are at a large table, they need to cluster together. Try giving only one copy of the worksheet to each group, not one per person, so they have to share. If these students cannot work together while seated, send them to work at a blackboard.

Sometimes you can get them to engage with each other converstionally. You may at times need to tell them explicitly "Janet has found what she thinks is an answer but Steve and Brenda apparently don't understand what she did, so Janet, you're going to have to explain it and see if you can convince Steve and Brenda." This is the most explicit you will ever want to be. Before this, perhaps try asking Brenda what her group has found so far, and don't let anyone else answer for her. If she says she's stuck, ask if her whole group is stuck, and if not, tell her you'll come back in 3 minutes and ask her again for a summary of what her group has done.

Sometimes you can give them specific tasks: Jason, finish the calculation you're doing; meanwhile Ann will add up Rob's numbers and will then check to see if they agree with yours; if not, it's up to all of you to judge which method if either was correct and why. If a group really has bad chemistry, change it. I've had students say to me: I just can't work with Judy - she won't listen and hogs

the discussion. In that case, put Judy in with someone smarter or more aggressive than she. When re-forming the groups, I usually randomize again, but if there are trouble students, sometimes I stack the deck so that they get put with students who can handle them.

2.4 Free riders

There are always lazy students who are content to let others do the work. If it's just laziness, I don't hesitate to reprimand them explicitly: "Adam - you're just staring into space and letting the other three figure out the problem; if the problems aren't challenging enough, then I can let you work faster in a group by yourself, but judging from your homework that's not the issue." On the other hand, if it's a student with a confidence problem who needs some nurturing, it's probably better to make a note of it and to continue to ask that student to explain what their group has done whenever you come around; note: you have to ask other students sometimes, or it gets too obvious. Involve the under-confident student as much as possible, with questions that are at their level but not patronizing. Also impress upon the others that it's their job to make sure the free rider is keeping up with the group since that person (you've just decided) will be in charge of the first group writeup. If a significant portion of the grade is based on exams (individual of course, not group), that will motivate students not to free-ride.

2.5 Staying on task

The less you have to reprimand here, the better. Make sure that when you tell them to get into groups they know exactly what they're supposed to start working on. Make the rounds quickly at the beginning so they don't start chatting, and keep an ear out for it later on. If a group continues to be bad about this, you can watch them, visit them more, chastize them humorously, make sure they don't get grouped together next time, etc., but if the whole class is bad, you should examine what you're doing that promotes doing something other than the math. You could be leaving them in small groups too long, while the slow groups finish. You could be joking with them too much during class time. I do also use reprimands, but sparingly - on Day 3 this year I was in an impatient mood so I reprimanded the two groups that took more than a minute to form their groups and start working. I said "Sounds interesting whatever you're talking about, but you've got to get started on problem 1 - time's short this week!"

2.6 Students who are behind

First, it's a good idea to know what you do for them and what you can't. You can do a lot for these students in office hours, but it's not realistic to be able to spend even as much as half an hour a week outside of class with any one student. So if they need help on a regular basis, suggest that they arrange for tutoring. You should find out before the semester what such resources are available. If they're doing passing work, but still underconfident, point out to them that if they continue to work at this level, they'll pass the class. Don't however make promises you can't keep about their grades; it's best not to prognosticate before they've taken an exam.

All that being said, you have to make sure they get the most out of their group work and don't drag down the group. They are as much afraid of this as you are. Here are some ways to build their confidence and make them more likely to participate to advantage. Spend some time around their group and be ready to pounce on those times when the lagging student, "Lenny", comes up with a good idea. Assign credit: if Jocelyn figures out how to do something with Lenny's idea, then it's "Lenny & Jocelyn's method". Try to give them constructive comments on their homework (I try to do this for everyone, but when time does not permit this, concentrate on students who need it). Perhaps select Lenny to give a presentation, either solo or on behalf of his group. On a one-time basis you can invest a little extra time to help him with this to make sure it goes over well (have him practice it on you till he's confident enough to field questions). Making sure someone is keeping up with their group is delicate - asking them to explain to you where their group is will help if they're not too far lost. Once they are far behind, this may not work; consider placing them next time with a group of students that talk a lot, don't go all that fast, and are as kind as possible. At worst, you may have to settle for this student participating minimally during class and trying to catch up on his or her own at home.

One thing you should avoid is to ask Lenny questions that are visibly way too easy. This will make him feel like he must be dumb. These questions are scary in general since the reward for a correct answer is almost zero and the penalty for an incorrect answer is large. At your discretion, you might chose Lenny to answer questions that are not black and white, asked of the class at large: which of these problems was the hardest?, what kinds of thinking did you think were necessary on this problem?, and so on. This does not in itself teach Lenny the material, but it keeps him from feeling like a ghost and reminds him that his opinion matters.

2.7 Students who are ahead

Having such a student can be a real boon if they are gifted teachers as well. If they have a good feel for how to explain things and help others, they will make your class run more smoothly than you can on their own. Even in this case, **avoid treating them in front of the class as a reliable source for right answers**. You don't want to create a situation where calling on them is tantamount to telling the class something yourself. Critical assessment is hard to teach. It is difficult to incorporate formats in which the students have to think things through, when we'd like to just tell them the right answer and get on with it. Calling on a reliable advanced student allows you to cheat on this, deceiving yourself and undermining your course goals. It is OK though, to treat an advanced student as a reliable source for intelligent commentary.

If a student is obviously head and shoulders above the rest from Day 1, you may consider exempting them from the course. That is, if they can do the worksheets on their own then they can probably pass the exemption exam, go on to the next course, and leave you with a more homogeneous class. Recommend that they see the course coordinator for an exemption exam; everyone will benefit from this. Later in the semester this is less of a good idea, though I've done it.

Assuming "Einstein" stays in the class and is not self-policing, you need to keep an eye on Einstein's group to make sure that Einstein is not explaining things to the others before they have a chance to figure it out themselves. Let Einstein explain things at the board in situations where you know there will be some wrong or unclear stuff in the explanation. Make sure though, that you give Einstein as much encouragement for what was right and clear as you would another student. If Einstein is a loner and tends to work fast but not share with the others, that will probably work out fine. You can explicitly designate Einstein as a group of 1 next time, or simply allow a *de facto* group of 1 to form. Sometimes, you can try asking Einstein explicitly to figure out a hint to give the rest of the group as to how to proceed but that won't completely solve the problem for them. It will make Einstein summon up teaching skills that are worthwhile in general, so it's worth a try, but be aware that Einstein may not be capable of this. In any case, don't let students disparage themselves in comparison to Einstein. You can say, "I see, because Einstein solved this problem in 5 minutes and you can't, you're going to give up?"

2.8 The physical classroom and group formation

Having discussed how small group dynamics are supposed to work, there are a few more words to say about how the groups should be formed. For example, why should the size be three or four? Two is too few, especially because college students have busy lives and do not always make it to class. [About that: it's best to have a grading policy that gives some credit for attendance. A good part of the learning is experiential, therefore a student who has learned all the skills that are testable in an exam setting still will not have gotten the whole course content without showing up and participating.] Five is too many because it is almost guaranteed that someone will hide. If every time you visit, any given student is talking on average one fifth of the time, it will take a very long time to detect which students are never talking and will be hard to correct even if it is detected. Some classrooms at Penn pretty much force groups of size six when enrollment is at or near capacity (the room is composed of tables that seat 6 each). In that case I have had to bust the sixes into two threes. These tend to merge back to one group of six unless you randomize seating daily, so that a group of three sits with a different group of three every day.

Even worse: you may be operating in a room not meant for active learning at all, either a theater meant for large lectures or a classroom with single desks that are clunky to move. The latter can be dealt with by moving them into groups every day. If you have to do this, try to make it a feature not a bug, assigning "desk monitors" each day, or making it a game who can convene their group first, or elaborating the set-up procedure, playing a theme song during set-up, whatever it takes! If you are unlucky enough to have a lecture amphitheater, there's really nothing you can do except get the administration to move you.

I already extolled the virtues of randomization. I do this religiously for the first randomizaiton. The second time, I modify if necessary so that no pair of students is together in both rounds one and two. Usually this is my only tampering, but occasionally I intervene to make sure any problem from round one is not present in round two. For example, if a particular students was intimidated by a group member, I don't put that student with another intimidating student. By the third randomization, I might be tempted to do more social engineering. Last time I taught the course my TA had strong feelings about trying this, so I let her formulate the groups.

3 Managing Socratic discussions

The hardest part of non-lecture teaching is conducting whole class discussions that are simultaneously productive, efficient and engaging. This is also the part that improves the most with practice. If you are new to active learning, you might want to incorporate Socratic whole class discussions only a little or not at all. Unless you are a natural at it, you should see it modeled before doing much of it yourself. When observing someone else's Socratic discussions, try to imagine what would have happened had they made different choices: told or not told the students something, came up with a good counter-question, decided to pursue or not to pursue a student's line of reasoning, etc.

3.1 Dead ends

When an idea is proposed, the instructor will usually know right away whether it will lead anywhere. If it won't, there is a strong temptation to discourage the students from pursuing the idea. This may be a mistake. Probably the best thing that can happen in a Socratic discussion is a *flaming dead end*, meaning that an incorrect line of reasoning leads to a consequence so patently false that the students are compelled to re-examine the road that got them there. If you see your students headed for one of these, then all you really need to do is encourage them to get there without undue delay. Some things you might want to do are: get them to explicitly reaffirm the wrong assumption, so that they will remember it later and be able to pinpoint it. In this case, shut down any further sidetracks that branch off of this one, e.g., "OK, that's a good idea, but first we'll finish pursuing this one." Hasten their demise by keeping the pace brisk, perhaps doing some of the arithmetic for them or providing clarifying paraphrases.

You can influence how flaming a dead end will be by making the issue more concrete: ask them to illustrate their result with actual numbers. For example, "So if the initial weight was 250 grams, then we see the final weight of $400 - 2w$ comes out to be what? Oh, I see, -100 grams; rather on the light side...").

Perhaps you will need to summarize their findings, juxtaposing two findings that are contradictory, or in the case that they have contradicted some of the given information, you may need to restate the givens by saying e.g., "so you have now proved that the only whole number between 100 and 500 having no two digits the same and satisfying blah blah blah is 337." To further rub it in, it often works well to insist that you believed their method and there must be some other mistake: "Ah, you've proved that the long division algorithm is wrong", or "Ah, you've proved that when you use

variables with subscripts, the usual laws of algebra don't apply" (if you can trust them to fight back on this one). Some of the techniques here that are adapted from the K-12 classroom may still work at the college level: joking, teasing, prodding, challenging. You will get to know what your students respond to and what they don't.

Other kinds of dead end are less useful. Perhaps they are following a reasonable line of inquiry but it doesn't get them anywhere: looking for a nonexistent pattern, introducing too many variables, classifying according to an ineffective scheme. A reasonable goal in this case is to get them to figure out that they're stuck. If you tell them (or indicate in any of 100 nonverbal ways) that their idea won't work, they will learn to look to you for validation of their ideas, whereas if they reach a dead end themselves and consciously decide to look for another approach, they have learned something valuable about problem-solving. That being said, there are ways to reduce the amount of time spent following a dead end. One trick is to decide after hearing a suggested approach whether to follow it immediately or whether to treat it as one of many to be written on the board before the class decides which to follow. If Jenny reports finding a pattern starting $4, 6, 8, 12$ and reports her reasoning as to what is likely to come next, you get to choose between (a) getting the whole class involved in speculating about the next number or (b) writing on the board "Idea: look for a pattern". The key feature of this trick is that you're not giving anything away. Approach (a) is reasonable in some contexts, where the discussion of pattern promises to have some depth, and more importantly, approach (b) is something you sometimes use when the approach offered is correct. In fact you should make sure to use (b) on occasions where there were multiple interesting approaches but the first one offered happens to be the best: you catalogue every group's approach before asking the class to pick one and follow it.

Another way to expedite matters is to insist that the goals be well defined. Often when a bad approach is put into words, it comes out sounding discouraging: we thought we'd name as many variables as possible and then hope that inspiration struck; we decided that if n was equal to 5 the solution was obvious but we don't know how to do it for any other value. Sometimes mild discouragement doesn't work. I remember a discovery style worksheet in a teacher prep course, designed to get the students to invent the binary number system by asking them to come up with a scheme for representing all numbers with 1's and 0's. This cost a full day of discussion of the relative merits of various schemes, none of which had anything to do with binary. The instructor was very successful that semester, and in my opinion the investment of that day paid off when students continued to work hard throughout the course because they didn't feel that the instructor was going to provide the answer for them. This takes guts, and doesn't work too well if the instructor conveys a growing uneasiness about the whole project. So if you're not up to following the wind,

you're probably better off cutting your losses, as in the next paragraph.

The least promising dead end is a total lack of ideas. Probably it's best not to convene a discussion at this point but to continue working in small groups where you can ask questions that elicit further work and break the impasse. But suppose a class discussion on a certain problem fizzles out midway. This might be a good time to drop it. If it's not essential that they end up knowing how to do the problem, and they don't have a realistic shot at finishing it for homework, perhaps make it into an extra credit assignment. If it is essential, consider dropping it for now. If you have recitations, this is a good time to relegate the topic to the TA. Don't get fixated on your ability to do it yourself, just be glad you have a TA. If not, try writing a worksheet for the next class that will lead them to it in more manageable steps, or stick it in a mini-lecture at the beginning of the next class. You probably need more time to solve this problem than you have on the spot.

3.2 How to listen

You need to listen to students and they need to listen to each other. Tom Lester once told me of a study showing that the average amount of time between when a teacher asks a student a question and when the teacher prompts the student or gives up on them is 2 seconds. Two seconds is longer than it sounds, but nowhere near long enough to formulate a coherent thought unless you were already thinking it before the quesiton was asked. There may not be anything you can do about the sound-bite trend in TV reporting, but there's a lot you can do about it in your classroom. The first thing to try is waiting. Don't nod yes or no, or say uh huh, or give the student any feedback at all until they have finished saying what they wanted to say. Then wait five or ten more seconds. The odds are that the student will, after pausing for breath, realize that they are not finished and continue. If not, at least the other students will have had a chance to think about what they just heard. If you're uncomfortable with this long a pause, try pacing or holding eye contact with the respondent as if you expect them to continue, or act as if you're trying to digest what they've just told you. In fact often you really will need time to think. If they said something that was wrong in a puzzling way, see if you can figure out what they really meant. Students will only listen to each other if you set an example, so make sure you don't respond without having really heard.

Students are also more likely to listen to each other if they feel that they are responsible for having understood it. In small groups they are more likely to feel this, but at a ratio of twenty or thirty to one, many feel that they can just take notes and sort it out later or not at all. They may also feel they have no right to interrupt since everyone else obviously understands. You can counter

this by demonstrating an expectation that each student understands what each other student has said. After one student says something the slightest bit unclear, ask another to repeat it in her own words. This is a good time to pick on students rather than have them raise their hand to volunteer a paraphrase. If student B can't paraphrase what student A said, it's not necessarily student B's fault. Student B can ask student A to clarify if necessary, or ask for volunteers for someone else to clarify. Make sure to go back and find out whether student C's clarification of student A's remark did in fact help student B. After a little experience you'll know better when to go through this routine. If student B simply wasn't listening, they might feel reprimanded, but that's OK. It doesn't really work when the remark was clear in the first place, although it doesn't hurt to get a quick affirmation from the whole class that it is clear so far. The basic standard you are setting is that the discussion involves the whole class and is not a collection of one on one dialogues between the teacher and individual students.

3.3 Staging

Your expectations of the nature of a class discussion are conveyed nonverbally as much or more than verbally. Many instructors ask the students to rearrange their desks into a large U shape for any but the briefest class discussions. A subtler but important technique is to put as much of the class as possible between you and the respondent. If you call on a student on the left side of the room, walk over to the right side as you're doing so. As the words flow between you and the respondent, the almost physical presence of a stream flowing between the two of you will wash over the students in between. You and the respondent also keep eye contact with the rest of the class this way.

It is often a good idea to get students to come up to the blackboard. Students will give longer monologues at the blackboard, so be prepared to be a more active moderator if the student is losing the rest of the class. Be careful not to make having the right solution a pre-requisite for coming to the board, lest the students stop thinking critically and accept any blackboard demonstration as a surrogate for your telling them something. When a student is at the board, I try to take up a position in the back or on the perimeter of the room. Sometimes I sit in the student's seat. This has the effect of including the rest of the class, as above, and also gives me a new vantage - you'd be surprised at what you see this way.

Other body language to be aware of is whether you are passing judgment on what you hear. Do your eyes flit impatiently with wrong answers? Do you gesture in agreement with right answers? Do you angle your body to the board as if to write down something correct, then pause if it's not what

you wanted? If you've chosen to incorporate Socratic discussion, it's because you want the students to develop their own judgment, so avoid this kind of tip-off.

A related topic is the use of intentional errors. These are a hit with kids, in a slapstick sort of way, but adult students tend to feel patronized. Instead, I substitute the mischeivous lie. If a student tells me the found all five regular polyhedra I may say, "Ah, so you haven't found the other two then?" They can often sense that I'm putting them on, but will still take the bait and try to prove that there aren't any more. On a problem best solved by assigning a variable to a certain quantity, I once told a group of frustrated students that I'd tell them the value of one quantity for free if they could decide which quantity they wanted to know. I planned to lie and tell them it was 10 when it was really 6. In fact I had tried this previously with success: the students figured out that the assumption of 10 led to a contradiction and were able to figure out the unique value that didn't lead to a contradition. This time it was even more successful. By the time I came back to give them their free question, they had figured out what quantity they wanted to be told, had put in x for this, and had gotten the solution (well ahead of the rest of the class).

3.4 Asking the right questions

When my teaching is evaluated by my students, they often say that I never answer questions, or answer them with another question. I take it as a complement even though it isn't meant as one. However, there are productive and relatively unproductive ways to do this. For example, this isn't terrible but is less productive than the second version:

> Student: Is this right?
> Instructor: I don't know. Does it sound right to you? Can you elaborate?
> **Better response:** Are you asking whether your computation is correct, or whether it will prove useful?

Here is another example where again both responses throw the question back at the student but the better response does so in a more specific way.

> Student: What should we do from here?
> Instructor: What do you think? Does anyone have any ideas?
> **Better response:** If you're wondering whether we can assign the variable z to be the

average of all prices, the answer is yes, but you haven't yet said whether we know anything about z.

When observing other classes, this is where you should let your imagination run free. Imagine what questions they might have asked. Your hindsight now will be your foresight tomorrow.

The question "do you understand?" is the most often abused. This is virtually the only question in the repertoire of the conventional lecturer and rarely elicits an honest response. Some better variants are: can you say that in your own words? could you do what John just said with different numbers? do you agree or disagree? in what way is this similar to what so and so did? These are all listening-comprehension questions, which will tell you whether a students has taken in the meaning of what was said more reliably than if you ask the student to report a Yes/No answer.

Good questioning can help to reach flaming dead ends. Ask what happens when $x = 5$, or whether their purported method works for all starting data and not just what was given. If a student gives a vague definition, find a borderline case and ask how their definition applies in that case. Try also questions that goad by disingenuity. If their method is more general than they realize, pretend they got lucky (ham it up!). Ask how they got lucky enough to try their method on a square rather than a pentagon or hexagon for which it "probably wouldn't have worked"; summarize for them that "because n was a multiple of 10, you were able to find k and l to fit the information given."

A final point to emphasize is: **don't make your questions too easy**. Often instructors who are having trouble getting students to speak up will tone down their questions, and are then puzzled that the chilly atmosphere doesn't improve. A timid student sees a too-easy question as a no-win situation. Either the student gets it right, which is not really impressive, or gets it wrong which is doubly embarrassing. It's better to make the reward higher. Long-term confidence is built by the realization that you can answer something nontrivial. And don't forget there may be students who are being quiet because they're bored; they will certainly appreciate your finding an alternative to going slower.

There are ways to construct nonthreatening questions that are suitable for tempting the timid out of their shell. Ask them to guess something that they don't have much of a basis for guessing yet, and get half a dozen different answers on the board to emphasize that it's still guesswork at this point. Ask others to agree or disagree with any of the guesses and discuss their relative merits. Ask them to help you read each other's mind: "Ah, 16 is an intelligent guess - can you see why Jackie thinks the answer might be 16?"

3.5 Order versus chaos

Ideally your students will be eager to answer your questions and discuss their ideas, but will listen patiently and attentively to each other and to you. If students are not willing to speak up and discuss their ideas, you need to loosen them up. It is a bad sign, for example, if the students are not happily chatting away when you enter the room five minutes before class, and are sitting in silence or whispering. In this case, you have probably done too well at eliminating chaos. Try assigning an activity in small groups where different groups are doing different things and they need to walk across the room to share information with each other. For example, there's a worksheet in my calculus class on infinite series where they approximate numerically some infinite sums and try to form conjectures based on each other's conclusions. Assigning a group project where students have to work outside of class together can tighten the bonds and make people feel more comfortable talking. When leading class discussions, be freer and more willing to follow the students ideas wherever they lead. Dispensing with handraising and having students just call out can quicken the pace. So can calling on students law-school style. If you can provide a supportive atmosphere for their answers - not being phony about what a nice try their wrong answer is, but being able to isolate and bring out the grains of truth in what they said - then this can be just the medicine for shy students.

A classroom that's too chaotic is a problem also. If you have to call the students to attention more than once before they pipe down and listen, or if there is crosstalk during class discussions, you probably ought to do something about it. You can address this explicitly, asking the students to pay attention to you and to each other. You have to be consistent about this or they won't believe you mean it. Indirect methods of dealing with this are, however, usually more effective and should be tried first or at least in parallel. Insist on an orderly formation of desks into a U shape before a class discussion, rather than having them minimally pertutrb the small-group seating arrangements. In a classroom with tables half occupied, ask them to reseat to completely occupy the tables nearest the front. It takes an extra minute, but it's worth it. In fact tell them you're pressed for time so they have to rearrange the desks in 30 seconds. A snappy set change will set the tone for what follows. When you observe crosstalk, try to get one of the crosstalkers up to the board to explain something, or to comment on what's just been said. By maximally involving that student in the lesson, you'll eliminate most of the off-task crosstalk, and the on-task crosstalk can probably be lived with. Another chaos reduction technique is to give them a more rigid idea of the structure of each class. Say you're going to spend 16 minutes in groups before convening a class discussion and then stick to it with absurd precision. The more aware they are of the structure of the lesson, the more they will stick to the tasks at hand.

The main point of this section is that you should make a conscious effort to optimize your position on the order-chaos axis, and that increasing order or chaos in the physical arrangements or chronological structure or types of assigned activities can help you change the balance in your class discussions.

4 TA management

At Penn, TA's are selected for active learning courses based on preferences (usually) but not ability (although rookie instructors tend to get paired with good TA's). Therefore, you will often find that your TA is a blank slate when it comes to how to teach in an active learning setting. I find training my TA to circulate effectively among students working in small groups to be very challenging. I intend to have them shadow me and *vice versa*, modeling for them and critiquing them the way instructor peer groups do for each other. However, resources are usually stretched thin enough that this is possible only very occasionally. Also, if it is too obvious that I'm teaching my TA how to teach, it will undermine the TA's standing with the students. Instead, I try to suss out what's going on with an individual worksheet in time to have an impromptu pow-wow with the TA a few minutes into the worksheet, to explain what I am seeing and how I plan to deal with it. As I am approaching, I can sometimes steal a few seconds of observation, and after the pow-wow I can sometimes ensure that the TA and I are in the same location for a while so that a little modeling can take place. Sometimes you have to settle for building a short meeting into the schedule after each class, or perhaps once a week.

To be honest, the biggest single problem I have had with TA's, is when they don't circulate. Usually it is because they feel awkward and don't know what to do. The longer you let them slide, not interacting with the students, the harder it will be to combat this.

Depending on how your course is set up, you may have a grader instead of or as well as an in-class assistant. Often I have had graders selected from the previous year's class. In this case I am thankful because they understand the type of grading that's required: how much they need to comment versus mark right/wrong, what writing standard to enforce, and so on. Some active learning classes have the feature that homeworks can be hard for the grader; even graduate student graders find some of the freshman calculus homework difficult and get it wrong the first time they solve it. I always have a 30 to 40 minute meeting with my graders before they begin to grade any homework set. In this session I check that their understanding of the material is correct and that they see through the problem, not just to an *ad hoc* computation that works but to the math beneath. Then we discuss what we would like to see from the students and how much we will insist on seeing it.

5 Curriculum

5.1 A vision

Those of us who have invested many hours inventing and plagiarizing the worksheets and problem sets for active learning courses have an advantage when it comes to day-to-day curricular decisions. We know where we're trying to get, and if something isn't working, we know whether we need to push it, to try another tack, or whether we should just drop it. The rest of this subsection gives advice for instructors who don't own the course and are not in a position to spend an extra day on some material, choose to skip some material, add a few hours to homework, and so forth.

One thing that helps is to thoroughly acquaint yourself with each worksheet before the semester begins. You will find yourself making decisions that are a lot better informed. You might substitute an easier worksheet once or twice where you know your students aren't up to learning the ambitious content in the materials you have been given. You might plan to get an early start on one worksheet, anticipating finishing the previous one early. You will know exactly what follows on what, and where the light is at the end of a tunnel through particularly difficult material.

The only way I know to do a sufficient job of familiarizing yourself with material is to personalize the coursepack. Plan to substitute, say, 10% of the material with worksheets where you think you see a better way to go, or see gaps needing to be filled in one place and worksheets leading nowehere that can be pruned somewhere else. Half the time you'll probably be wrong, but the net effect will be positive nonetheless.

Personalizing the coursepack, you will develop (if you haven't already) your own vision for the course. What are you trying to accomplish and what is the role of each section of the curriculum? If you don't have an idea of this, you will continually encounter awkward moments in class where you're trying to get students to understand something and start feeling like you're just making them jump through hoops. I came across that when teaching someone else's active learning BC calculus course during a long section on integration techniques and another long section on infinite series. For the infinite series, looking at the course more deeply made me appreciate why the material had to be there, and I was able to convey it to the students. For the integration techniques, I never did see that point. This was not helpful at the time but later led to a useful restructuring of the curriculum.

Once you have firmly in mind not only the underlying concept but what about it they really need to know, you'll be able to devise a new approach on the fly (or more honestly, if you don't then you

certainly won't). When confronted with an unfamiliar pedagogy such as active learning, students are more likely to question openly why they're being made to do what you ask them to do. You will have trouble answering if you don't have your own vision for the course. If you are reading this because you are a Penn instructor, then we hope we've created enough raw material so that personalizing the coursepack won't take anywhere near the time it took to create the pilot versions.

One last remark about your vision for the course: students may be enrolled for a variety of reasons. It could be extremely helpful for you to survey them on the first day (or, these days, send an online survey the week before) to find out why they're enrolled and what they're hoping to get from the course. You might hear something that changes your vision.

5.2 Pacing

Another advantage we had during the pilot versions is that we were defining the expectations as to what could be covered in a semester, and so automatically lived up to them. Now that there are norms for the course, there is pressure to keep up with them. If you find you're going "too slowly", it's worth figuring out why, but you should resist the temptation to hurry. The course was designed the way it was in order to promote certain goals. If you start lecturing, forcing ideas on the students, letting the fast ones serve as surrogate expositors, and so forth, their rate of learning will go down, not up. You just won't know it right away. Instead, consider the following alternatives. (1) Perhaps you've got a slow class. In this case you should probably go at the pace they need and cover less. They'll be better off in the long run. But if this happens to you again the next time, try a different tack! (2) You may be underassigning homework. If you ask them more often to finish trying to understand a problem sheet on their own at home and hand it in for a writeup, they will do it. You'll have to be careful to stay in touch with their needs, but if you assign the homework for a date that allows them to come back and ask for more class time or office hour time when needed, they should be able to handle the responsibility. (3) You may be covering the material more thoroughly than intended. When I write a worksheet, I often include extra, more difficult problems at the end that I plan to use only if time permits (which it seldom does). Use your judgment as to what portion of a worksheet is "par for the course" and compare with other instructors. (4) Your efforts to be Socratic might be too extreme. Deciding whether you're telling the students too much or too little can be like figuring out whether a plant is over- or under-watered. You'd best check with an expert.

As in proper questioning, a good worksheet needs to be nontrivial. If students are going through entire worksheets ahead of schedule, probably they are not being challenged enough (although this is

a great morale booster when it happens occasionally). If you're constructing a significant amount of new material for your course, recognize the warning signs of too easy material, since they're subtler than warnings about the opposite. A general loss of interest, low variance in homework scores, a lack of good places to begin class discussions, all indicate that either your materials are not meaty enough or that you're doing too much of the students' work for them in class.

5.3 Resources

This subsection pertains strictly to Penn active learning classes. For Math 110 (business calculus), we hope to have in your hands a very complete set of materials. This includes a day-by-day schedule, lecture notes for each day, worksheets for each day, homework, specific reading and online self-check assignments, study guides, extra problems with solutions for exam preparation. There will also be (ask if you don't see it) a copy of the daily lesson plans written by a recent instructor. These will comment on how each worksheet has worked (or failed) in the past, what the students got out of it, how the discussion went or should have gone, what advance preparations are advisable, and so on. There is only one version of the course and we expect first time instructors to stick to it. All of this is available on the faculty server.

For Math 103 and 104 (AB and BC calculus respectively) there are at least two different sets of course materials posted on the faculty server. At least one set (the less Socratic version) will have concepts sequenced in a detailed way, similar to programmed instruction. The other set will be more discovery oriented and flexible. At the time of writing, I don't know whether the 103-104 materials will have come with instructors' notes. Past exams with solutions are available, and these serve as exam preparation aides.

6 Organization

Active learning classes are inherent more disorganized than traditional classes, which is why you need to pay particular attention to organization.

6.1 Your records

Students, react positively to the appearance of organization. Probably The pre-service teachers I taught at Wisconsin were particularly attuned to this because they are always conscious of their dual role as student and future teacher, and therefore tend to put themselves in the place of their instructor, judging you on your pedagogical techniques and on technical aspects such as organization, legibility and so forth. **Be a compulsive record-keeper.** I find it very helpful to reserve the 20 minutes after each class for writing a short summary of what happened in class that day. That's also where the lesson plan advice for next year's teacher comes from. If I've told a student I'd find them an extra-credit assignment, or if I've promised to bring something to class next time or promised that the next class woul begin with a discussion of something, I can write this down along with other notes as to what I have in mind for the next class, and I'll see it five minutes before the next class when I look at my notes. In an active learning class there is a greater opportunity for unexpected things to happen, therefore a greater need to write down what did happen.

Keeping date records of the homework you've assigned, both due dates and the date it was assigned, is essential. The use of Canvas makes this automatic unless you deviate from what you intended at the start, and even then, announcing it on Canvas will give you a record of the change. Whether or not you accept late homework is up to you, but it is certainly better to have students ask solicitously in advance for you to accept their late papers, which you will probably grant, than for students to assume it's OK and be upset if you don't grant them an extension after the fact. They are more likely to do the latter if they get the idea that you yourself don't remember when the homework was due. In fact, if I arrive at class early, I often take the opportunity to write up a reminder of what is due in the upcoming week or two.

6.2 Grading

The message here is the same as in the previous section. Include in your coursepack a clearly defined grading policy, specifying what portions of the grade are from homework, exams, quizzes, projects, group work, attendance, or whatever else you grade on. Give it enough thought so that you remember it easily, and can answer their questions immediately. If historical grade distribution data is available, you would be well advised to stick to it, since this will help to allay fears that the unusual pedagogical style will adversely affect their grade. I suggest that you announce a policy that's a little stingier than you think will be your bottom line. This allows you increase some grades at your discretion. One way I sell students on active learning is explain that I will get to know them

far better than in a lecture course, and can therefore make sure their final grade is not less than they deserve.

If you find yourself spending too much time grading the students' assignments, then you're in good company. They learn a lot from this, so I encourage you to do as much as you can stomach in the way of grading writeups. That being said, there are ways to reduce the workload a little.

Some instructors grade the finished worksheets in some form. In the teacher prep math courses I taught at Wisconsin and OSU, students had to write up nearly every worksheet, not they way they stumbled through it in class but the way they understood it after class discussion and explaining their work to others or being on the receiving end of such an explanation. This made sense for a course in which the process and the verbalization of mathematics represented 80% of what was to be learned. To reduce the load I assigned writeups to be ready for collection two class sessions after being completed in class, but let the students know that it would be a day-to-day decision whether a particular worksheet would actually be graded. That way I could quickly look over the work each time and see on which assignments feedback would be most helpful. If you do this, it's best to bite the bullet the first few weeks and grade everything, both to set the expectation that these count and to give early feedback as to what kind of writing you expect.

Every now and then you can assign a group assignment instead of an individual assignment. Don't do this more than once between each change of groupings, and make sure the assignment is appropriate for a group writeup. Realize that it is hard for students to find time to meet outside of class, so ideally the assignment should combine what they were able to work out together in class with some labor they can easily divide and do at home.

In my calculus courses I do not grade the worksheets. I give weekly homework assignments that are pretty hard, on which they are allowed to collaborate but must hand in work written up individually. I am fortunate enough to have a grader in calculus, otherwise this would be somewhat onerous. Still, I have to instruct my graders on how to give quality feedback without working far longer than they are paid.

Another useful tactic is multiple choice questions on exams. I always include multiple choice or True/False sections on exams, because the purpose of these is evaluation and not instruction. These tend to add noise to the grade, however, if they are graded as all or nothing. It always amazes me that instructors put "distractors" on exams: wrong answers that are very close to right, ones you would get if you did the whole problem right but forgot a factor of 2. To my mind, some wrong answers are way better than others and should be scored accordingly. Also on MC or TF questions,

I always tell the students to provide justfication if they want to be eligible for partial credit. And of course there is a limit on how much TF and MC you can use on an exam in a course that stresses reasoning and problem solving. You need to save room on the exams for questions where more thorough explanations are demanded.

6.3 The bell

When the bell rings at the end of class, everything becomes exponentially harder. Even in a building without synchronized bells, students become very restless when the class period ends. My advice is to watch the clock like a hawk, so that you can make sure to wrap up the discussion at 2 minutes before the end. The discussion usually leaks over an extra minute, giving you one minute to say any summary comments or give instructions on homework, etc. This can be an important routine even when you have little to say: it makes you seem organized and on top of things.

If you can tell you are going to want to go overtime, because of a red-hot discussion you want to complete or something that's necessary so they can do their homework, announce to them 5 or 10 minutes ahead of time that you will probably be going overtime. It's best if you've let them go a minute or two early once before and have mentioned at the time that you're banking those minutes for such an occasion as this. If they're working in small groups at the end of class, it's less crucial but you still may want to halt them 1 or 2 minutes before the bell for closing remarks - it's even harder to get their attention after the bell when you don't already have the stage.

In short: **don't ever be surprised by the bell**.