1. Let $f(x) = \frac{60}{x}$. Approximate the integral $\int_1^5 f(x) \, dx$ using:

(a) The midpoint rule, with 2 intervals,

The interval over which we are integrating is $[1, 5]$, so the width of each subinterval is $\Delta x = (5 - 1)/2 = 2$. We therefore split the interval $[1, 5]$ into the two subintervals $[1, 3]$ and $[3, 5]$. The midpoints of these intervals are 2 and 4, so the midpoint rule gives

$$\int_1^5 f(x) \, dx \approx \Delta x [f(2) + f(4)] = 2 \left( \frac{60}{2} + \frac{60}{4} \right) = 90.$$  

NB: don’t confuse the midpoint rule with the trapezoidal rule.

(b) Simpson’s rule, with 4 intervals.

In this case, $\Delta x = (5 - 1)/4 = 1$. Therefore the points at which we should evaluate the function $f(x) = 60/x$ are 1, 2, 3, 4, and 5: $f(1) = 60$, $f(2) = 30$, $f(3) = 20$, $f(4) = 15$, $f(5) = 12$. Simpson’s rule therefore gives

$$\int_1^5 f(x) \, dx \approx \frac{\Delta x}{3} [f(1) + 4f(2) + 2f(3) + 4f(4) + f(5)]$$

$$= \frac{1}{3} (60 + 4(30) + 2(20) + 4(15) + 12)$$

$$= \frac{292}{3}.$$
Wednesday Quiz 3
Maths 104 - Calculus I
February 23, 2011

Note: In order to receive full credit, you must show work that justifies your answer.

1. Evaluate
\[ \int \frac{3x^2 - 5x + 6}{x^3 - 2x^2} \, dx. \]

This is very similar to Monday quiz 2, so I will solve for the constants in a different way.

Since the degree of the numerator is strictly less than the degree of the denominator, we don’t need to divide the numerator by the denominator.

The next step is to factorize the denominator: \( x^3 - 2x^2 = x^2(x - 2) \). We should therefore be able to find constants \( A \), \( B \) and \( C \) such that
\[
\frac{3x^2 - 5x + 6}{x^3 - 2x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x - 2}.
\]

By multiplying both sides of the equation by \( x^3 - 2x^2 = x^2(x - 2) \), we see this is equivalent to
\[
3x^2 - 5x + 6 = Ax(x - 2) + B(x - 2) + Cx^2.
\]

The right hand side is
\[
Ax^2 - 2Ax + Bx - 2B + Cx^2 = (A + C)x^2 + (-2A + B)x - 2B,
\]
so by comparing coefficients with the left hand side, we see that \( A + C = 3 \), \(-2A + B = -5\), and \(-2B = 6\). The last equation gives \( B = -3 \), so plugging this into the second equation gives \(-2A - 3 = -5\), so \( A = 1 \), and then using the first equation we get \( C = 2 \). Therefore
\[
\frac{3x^2 - 5x + 6}{x^3 - 2x^2} = \frac{1}{x} - \frac{3}{x^2} + \frac{2}{x - 2},
\]
and so
\[
\int \frac{3x^2 - 5x + 6}{x^3 - 2x^2} \, dx = \int \frac{1}{x} - \frac{3}{x^2} + \frac{2}{x - 2} \, dx
= \ln |x| + \frac{3}{x} + 2 \ln |x - 2| + D = \ln |x(x - 2)| + \frac{3}{x} + D.
\]