Maths 240 homework for 2 August Due: 7 August

4.1 Find the area of region bounded by the loop in the curve \( y^2 = x^3 + x^2 \). [Hint: The curve can be parameterized by \( x = t^2 - 1, y = t^3 - t \)].

4.2 (Final exam, Fall 2011) Find the value of the line integral

\[
I = \int_C (3\pi x^2 y + ye^x)dx + (\pi x + \pi x^3 + e^x)dy,
\]

where \( C \) is the curve parametrized by \( x = \sin t, y = t \) for \( 0 \leq t \leq \pi \), and oriented in the direction of increasing \( t \).

4.3 Section 9.15, ex 24.

4.4 Find the volume of the region in the first octant bounded by \( z = 1 - y^2 \), \( x + y = 2 \), and the coordinate planes.

4.5 Evaluate \( \iiint_D x^2 + y^2 + z^2 \, dV \), where \( V \) is the region within the sphere \( x^2 + y^2 + z^2 = 4 \) and above the cone \( z = \sqrt{x^2 + y^2} \).

4.6 (Final exam, Fall 2010) Find the outward flux

\[
\iint_S \mathbf{F} \cdot \mathbf{n} \, dS
\]

of the vector field

\[
\mathbf{F} = 4xy^2 \mathbf{i} + 3yj + 4zx^2 \mathbf{k}
\]

where the surface \( S \) is the boundary of the region \( 1 \leq x^2 + y^2 \leq 4, 0 \leq z \leq 1 \).

4.7 Find \( \iiint_D (x - z^2) e^x \, dV \), where \( D = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 1 \} \) is the unit ball. [Hint: Consider the vector field \( \mathbf{F}(x, y, z) = (-z^2e^x, 0, xze^x) \)].

4.8 (Final exam, Fall 2011) Consider the vector field

\[
\mathbf{W} = x^3y^2 \mathbf{i} - x^2y^3 \mathbf{j} + (1 + z) \mathbf{k}
\]

Find the outward flux of \( \mathbf{W} \) through the portion \( S \) of the paraboloid \( z = 4 - x^2 - y^2 \) which lies above the \( xy \)-plane.