1. Find the length of the curve \( \mathbf{r}(t) = \langle t^2, 2t, t^3/3 \rangle \), \( 1 \leq t \leq 3 \).

\[
\mathbf{r}'(t) = \langle 2t, 2, t^2 \rangle, \text{ so } |\mathbf{r}'(t)| = \sqrt{(2t)^2 + 2^2 + (t^2)^2} = \sqrt{4t^2 + 4t^2 + 4} = t^2 + 2.
\]

Therefore the length of the curve is

\[
\int_1^3 (t^2 + 2) \, dt = \left[ \frac{t^3}{3} + 2t \right]_1^3 = \frac{27}{3} + 6 - \left( \frac{1}{3} + 2 \right) = \frac{38}{3}.
\]

2. A projectile is fired with an initial speed of 50 m.s\(^{-1}\) at an angle of 30° to the horizontal. If the projectile is fired at time \( t = 0 \), when does it land? (Use \( g = 10 \text{ m.s}^{-2} \)).

We may assume the motion of the projectile takes place in the \( xy \)-plane, and that the projectile is fired to the right, starting at the origin. Then the initial velocity vector (in m.s\(^{-1}\)) is \( 50 \cos(30°) \mathbf{i} + 50 \sin(30°) \mathbf{j} = 25\sqrt{3} \mathbf{i} + 25 \mathbf{j} \). The acceleration of the projectile (in m.s\(^{-2}\)) is due to gravity: \( \mathbf{a} = -10 \mathbf{j} \).

If \( \mathbf{r}(t) \) is the position of the particle at time \( t \), then \( \mathbf{a} = \mathbf{r}''(t) \), so

\[
\mathbf{r}'(t) = \int \mathbf{r}''(t) \, dt = \int -10t \, dt = -10t \mathbf{j} + \mathbf{c} \text{ for some constant vector } \mathbf{c}.
\]

We can work out \( \mathbf{c} \) by substituting \( t = 0 \):

\[
25\sqrt{3} \mathbf{i} + 25 \mathbf{j} = \text{initial velocity} = \mathbf{r}'(0) = \mathbf{c}.
\]

Therefore \( \mathbf{r}'(t) = -10t \mathbf{j} + 25\sqrt{3} \mathbf{i} + 25 \mathbf{j} = 25\sqrt{3} \mathbf{i} + (25 - 10t) \mathbf{j} \). Thus

\[
\mathbf{r}(t) = \int \mathbf{r}'(t) \, dt = \int 25\sqrt{3} \mathbf{i} + (25 - 10t) \mathbf{j} \, dt = 25\sqrt{3} \mathbf{i} + (25t - 5t^2) \mathbf{j} + \mathbf{d} \text{ for some constant vector } \mathbf{d}.
\]

We can work out \( \mathbf{d} \) by substituting \( t = 0 \):

\[
\mathbf{0} = \text{initial position} = \mathbf{r}(0) = \mathbf{d}.
\]

Therefore \( \mathbf{r}(t) = 25\sqrt{3} \mathbf{i} + (25t - 5t^2) \mathbf{j} \). When the projectile lands, the \( \mathbf{j} \) component is zero, so \( 25t - 5t^2 = 0 \), i.e. \( t(5 - t) = 0 \). Thus \( t = 0 \) or \( 5 \), but since \( t = 0 \) is when the projectile is fired, the projectile lands after 5 seconds.

Of course you could also start with the fact that the vertical component of the trajectory is given by \( \sin(\theta)t - \frac{1}{2}gt^2 \), but you should be able to derive this on your own.
1. Find the domain and range of the function \( f(x, y) = e^{\sqrt{1-x^2-y^2}} \).

The function is defined if and only if the expression under the square root is not negative. Therefore \( 1 - x^2 - y^2 \geq 0 \), so \( x^2 + y^2 \leq 1 \). Thus the domain is

\[
D = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}.
\]

Note: \( x^2 \leq 1 - y^2 \) is equivalent to \( |x| \leq \sqrt{1-y^2} \), and not \( x \leq \sqrt{1-y^2} \).

For the range, notice that \( 0 \leq 1 - x^2 - y^2 \leq 1 \), so \( 0 \leq \sqrt{1-x^2-y^2} \leq 1 \). Since the exponential function is increasing, \( e^0 \leq e^{\sqrt{1-x^2-y^2}} \leq e^1 \). Also note that given any real number \( r \) with \( 1 \leq r \leq e \), we can find \( (x, y) \in D \) with \( e^{\sqrt{1-x^2-y^2}} = r \). Therefore the range is \( \{r \in \mathbb{R} : 1 \leq r \leq e\} \), which we can also write as the interval \([1, e]\).

2. Describe the level curves of \( f(x, y) \). (You don’t need to draw them)

If \( f(x, y) \) is a constant \( k \), then \( e^{\sqrt{1-x^2-y^2}} = k \), so \( \sqrt{1-x^2-y^2} = \ln k \), therefore \( 1 - x^2 - y^2 = (\ln k)^2 \), i.e. \( x^2 + y^2 = 1 - (\ln k)^2 \). This describes a circle centered at \((0, 0)\) (for appropriate values of \( k \)). Thus the level curves of \( f(x, y) \) are a family of concentric circles.

Let \( f(x, y, z) = (y + z)^x \). What is \( f_y(x, y, z) \)?

Think of \( z \) and \( x \) as being constants. Then

\[
 f_y(x, y, z) = x(y + z)^{x-1} \frac{\partial}{\partial y}(y + z) = x(y + z)^{x-1}
\]

by the power law and the chain rule.