Math 312 Homework 1
Due Tuesday, July 3, 2018

Problem 1 (Strang 1.1.3). If $v + w = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ and $v - w = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$, compute and draw the vectors $v$ and $w$. Also draw $v + w$ and $v - w$. In a second picture, again draw $v$ and $w$ and shade in all linear combinations $cv + dw$ restricted by $c \geq 0$ and $d \geq 0$.

Problem 2 (Strang 1.2.7). Find the angle $\theta$ (from its cosine) between these pairs of vectors:

1. $v = \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$ and $w = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
2. $v = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$ and $w = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$
3. $v = \begin{bmatrix} 1 \\ \sqrt{3} \end{bmatrix}$ and $w = \begin{bmatrix} -1 \\ \sqrt{3} \end{bmatrix}$
4. $v = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $w = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

Problem 3 (Strang 1.2.13). Find a unit vector, $u_1$, in the same direction as (1,0,1). Find nonzero unit vectors $u_2$ and $u_3$ perpendicular to $u_1$ and to each other.

Problem 4 (Strang 1.3.4). Let

$$w_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, w_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \text{ and } w_3 = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}.$$ 

Are these vectors linearly independent or linearly dependent? If they are linearly dependent, give a linear combination (where not all of the coefficients are 0) of them that produces the zero vector. Do the three vectors lie in a line, a plane, or do they make of all of 3-dimensional Euclidean space? Is the matrix, $W$, with those three columns invertible?

$$W = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}.$$
Problem 5 (Strang 2.1.21). What 2 by 2 matrix $R$ rotates every vector through 45°? The vector $(1, 0)$ goes to $(\sqrt{2}/2, \sqrt{2}/2)$. The vector $(0, 1)$ goes to $(-\sqrt{2}/2, \sqrt{2}/2)$. Those determine the matrix. Draw these particular vectors in the $xy$ plane and find $R$. $R^2$ gives the matrix that rotates every vector by 90°. $R^4$ gives the matrix that rotates every vector by 180°, which is just $-I$. $R^8$ gives the matrix that rotates every vector by 360°, which is the same as leaving every vector fixed; hence, $R^8 = I$. 