Math 312 Homework 2

Due Thursday, July 5, 2018

**Problem 1.** Write the following system of equations as a matrix equation, \( Ax = b \).

\[
\begin{align*}
3x + y - z &= 5 \\
6x - y + z &= 12 \\
3x + 7y + 2z &= -26
\end{align*}
\]

Reduce it to a triangular system, \( Ux = c \), and then solve for \( x \) using back-substitution.

**Problem 2 (Strang 2.3.17).** The parabola \( y = a + bx + cx^2 \) goes through the points \((x, y) = (1, 4)\) and \((2, 8)\) and \((3, 14)\). Find and solve a matrix equation for the unknowns \((a, b, c)\).

**Problem 3 (Strang 2.4.1).** \( A \) is 3 by 5, \( B \) is 5 by 3, \( C \) is 5 by 1, and \( D \) is 3 by 1. All entries are 1. Which of these matrix operations are allowed, and what are the results?

1. \( BA \)
2. \( AB \)
3. \( ABD \)
4. \( DC \)
5. \( A(B + C) \)

**Problem 4 (From Strang 2.5.10 and 2.5.25 and 2.5.31).** Find the inverses of the following matrices:

1. \[
\begin{bmatrix}
0 & 0 & 0 & 2 \\
0 & 0 & 3 & 0 \\
0 & 4 & 0 & 0 \\
5 & 0 & 0 & 0
\end{bmatrix}
\]

2. \[
\begin{bmatrix}
2 & 1 & 1 \\
1 & 2 & 1 \\
1 & 1 & 2
\end{bmatrix}
\]

3. \[
\begin{bmatrix}
1 & -1 & 1 & -1 \\
0 & 1 & -1 & 1 \\
0 & 0 & 1 & -1 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
Problem 5 (Strang 2.6.7). What three elimination matrices \( E_{21}, E_{31}, E_{32} \) put \( A \) into its upper triangular form \( E_{32}E_{31}E_{21}A = U \)? Multiply by \( E_{32}^{-1}, E_{31}^{-1} \), and \( E_{21}^{-1} \) to factor \( A \) into \( L \) times \( U \):

\[
A = \begin{bmatrix}
1 & 0 & 1 \\
2 & 2 & 2 \\
3 & 4 & 5
\end{bmatrix} \
L = E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}
\]

Problem 6 (Strang 2.7.22). Find the \( PA = LU \) factorizations for

\[
A = \begin{bmatrix}
0 & 1 & 1 \\
1 & 0 & 1 \\
2 & 3 & 4
\end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix}
1 & 2 & 0 \\
2 & 4 & 1 \\
1 & 1 & 1
\end{bmatrix}
\]