Problem 1 (adapted from Strang 3.3.1). Let

\[
A = \begin{bmatrix}
2 & 4 & 6 & 4 \\
2 & 5 & 7 & 6 \\
2 & 3 & 5 & 2
\end{bmatrix}
\]

Find the rank of \(A\), a basis for the column space of \(A\), and a basis for the row space of \(A\). Let

\[
b = \begin{bmatrix}
4 \\
3 \\
5
\end{bmatrix}
\]

Find the complete/general solution to

\[Ax = b\]

Problem 2 (Strang 3.3.24). Give examples of matrices \(A\) for which the number of solutions to \(Ax = b\) is

1. 0 or 1, depending on \(b\)
2. \(\infty\), regardless of \(b\)
3. 0 or \(\infty\), depending on \(b\)
4. 1, regardless of \(b\).

Problem 3 (adapted from Stang 3.4.35). Let \(P_n\) be the vector space of polynomials of degree at most \(n\) with real coefficients. So, an arbitrary element of \(P_n\) looks like \(a_0 + a_1x + a_2x^2 + \cdots + a_nx^n\). What is the dimension of \(P_n\)? Find a basis for \(P_n\). Now consider \(P_3\), polynomials, \(p(x)\), of degree at most 3 with real coefficients. Find a basis for the subspace with \(p(1) = 0\).

Problem 4 (Strang 3.4.13). Find the dimensions of these 4 spaces. Which two of the spaces are the same? (a) column space of \(A\), (b) column space of \(U\), (c) row space of \(A\), (d) row space of \(U\):

\[
A = \begin{bmatrix}
1 & 1 & 0 \\
1 & 3 & 1 \\
3 & 1 & -1
\end{bmatrix}
\quad\text{and}\quad
U = \begin{bmatrix}
1 & 1 & 0 \\
0 & 2 & 1 \\
0 & 0 & 0
\end{bmatrix}
\]
Problem 5 (Strang 3.5.3). Find bases and dimensions for the four subspaces associated with $A$ ($C(A), N(A), C(A^T), N(A^T)$).

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 4 & 6 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

Problem 6 (Strang 3.5.7). Suppose the 3 by 3 matrix $A$ is invertible. Write down bases for the four subspaces for $A$, and also for the 3 by 6 matrix $B = [A \ A]$. (The empty set forms a basis for the zero space).