Math 312 Homework 8

Due Wednesday, July 25, 2018

Problem 1 (Strang 6.4.7). Find an orthogonal matrix $Q$ that diagonalizes

$$S = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & -2 \\ 2 & -2 & 0 \end{bmatrix}. $$

What is $\Lambda$?

Problem 2 (Strang 6.4.13). Write $S$ and $B$ in the form $\lambda_1 x_1 x_1^T + \lambda_2 x_2 x_2^T$ of the spectral theorem $Q \Lambda Q^T$:

$$S = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 9 & 12 \\ 12 & 16 \end{bmatrix}. $$

Problem 3 (Strang 6.5.2). Which of $S_1, S_2, S_3, S_4$ has two positive eigenvalues? Use a test, don’t compute the $\lambda$’s. Also find an $x$ so that $x^T S_1 x < 0$, so $S_1$ is not positive definite.

$$S_1 = \begin{bmatrix} 5 & 6 \\ 6 & 7 \end{bmatrix} \quad S_2 = \begin{bmatrix} -1 & -2 \\ -2 & -5 \end{bmatrix} \quad S_3 = \begin{bmatrix} 1 & 10 \\ 10 & 100 \end{bmatrix} \quad S_4 = \begin{bmatrix} 1 & 10 \\ 10 & 101 \end{bmatrix}. $$

Problem 4 (Strang 6.5.36). Suppose $S$ is positive definite with eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$.

1. What are the eigenvalues of the matrix $\lambda_1 I - S$? Is it positive semidefinite (all eigenvalues nonnegative)?

2. How does it follow that $\lambda_1 x^T x \geq x^T S x$ for every $x$?

3. What is the maximum value of $\frac{x^T S x}{x^T x}$ as $x$ ranges over all nonzero vectors?