1) Prove that for every non-negative real number $y \in \mathbb{R}$, and every $r \in \mathbb{N}_{r>0}$, there exists a unique non-negative real number $x \in \mathbb{N}$ such that $y = x^r$.

[Hint: Let $z > 1$ be fixed such that $zy > 1$, and for every $n \in \mathbb{N}_{n>0}$ let $m_n \in \mathbb{N}$ be such that $m_n^n \leq yz^{nr} < (m_n + 1)^r$. NOTE: Such an $m$ exists (WHY). Hence $(\frac{m_n}{z^n})^r \leq y < (\frac{m_n+1}{z^n})^r$. Then $\frac{m_n+1}{z^n} - \frac{m_n}{z^n} \to 0$ (WHY). Finally show that $(\frac{m_n}{z^n})_n$ and $(\frac{m_n+1}{z^n})_n$ are convergent to the same real non-negative number, say $\frac{m_n}{z^n} \to x$, and that $x^r = y$, etc. . . .]

Solve the following problems at the end of Ch.14 of the book *Mathematical Thinking* by D’Angelo & West (starting on p.287):

2) Exercises 2, 3, 4, 5.
3) True/ False: Exercises 8, 9, 10, 11, 12.
4) Exercises 16, 17
5) Exercises 18, 19, 20, 21, 22
6) Exercises 39, 60, 66