Math 114: Typical Problems Section 12

1. Consider the line \( L_1 \) which is parallel to \( y = x + 2 \) and passing through the point \((1, 2)\) and the line \( L_2 \) which is perpendicular on the line \( 2y - 2x = 1 \) and passes through the point \((3, 1)\). Answer the following:
   a) Compute the intersection \( L_1 \cap L_2 \).
   b) Does \( L_1 \) intersect the circle of radius 2 and center \((3, 2)\)?

2. Consider the line \( L \) which is parallel to \( \vec{v} = 2\vec{i} - \vec{j} - 2\vec{k} \) and passing through the point \((1, 2, -1)\) and the plane \( \mathcal{P} \) which is perpendicular on the vector \( \vec{u} = -\vec{i} - \vec{j} + 2\vec{k} \) and passes through the point \((3, 1, 0)\). Answer the following:
   a) Compute the intersection \( L \cap \mathcal{P} \).
   b) Does \( L \) intersect the sphere of radius 2 and center \((3, 2, 1)\)?

3. Compute the point on the intersection line of the planes \( x - y - z = 2 \) and \( x + y - z + 2 = 0 \) which is closest to the point \((1, 1, 1)\).

4. Which plane contains the diagonal in the \( xy \)-plane and is perpendicular on the plane \( x - y + z = 1 \)?

5. Find the distance from the \( z \)-axis to the intersection line of the planes \( x = y + z + 1 \) and \( z = 2x + 2y + 1 \).

6. Find the value of the \( x \)-coordinate where the plane through the points \((4, 1, 1), (1, 2, 1), \) and \((1, 1, 2)\) intersects the \( x \)-axis.

7) Describe the set of all the points in the \( xy \)-plane which have equal distance to:
   a) the points \((2, 3)\) and \((1, 1)\).
   b) the lines \( x + y = 2 \) and \( 2x - y = 0 \).

8) Describe the set of all the points in the \( xyx \)-space which have equal distance to:
   a) the points \((2, 3, 0)\) and \((0, 1, 1)\).
   b) the line \( \vec{r}(t) = (1, 1, 1) + t(-1, 0, 2), t \in \mathbb{R} \) and the plane \( 2x - y - z = 1 \).
   c) the planes \( x + y + z = 1 \) and \( x - y - 2z = 1 \).

9) Answer whether the following is true or false, and give a reason / counterexample: Let \( \vec{u}, \vec{v} \) be non-zero vectors in the \( xyz \)-space.
   a) If \( \vec{u} - a\vec{v} \) and \( \vec{u} + a\vec{v} \) have the same length for some number \( a \neq 0 \), then \( \vec{u} \perp \vec{v} \).
   b) If \( |\vec{u} + a\vec{v}| = |\vec{u}| + |a||\vec{v}| \) for some \( a \neq 0 \), then \( a > 0 \) and \( \vec{u} = a\vec{v} \).

   [Hint: What is the length of a vector in terms of dot product?]"
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