Math 114: Typical Problems Section 12

1. Consider the line L_1 which is parallel to $y = x + 2$ and passing through the point $(1, 2)$ and the line L_2 which is perpendicular on the line $2y - 2x = 1$ and passes through the point $(3, 1)$. Answer the following:

a) Compute the intersection $L_1 \cap L_2$.

b) Does L_1 intersect the circle of radius 2 and center $(3, 2)$?

2. Consider the line L which is parallel to $\vec{v} = 2\vec{\imath} - \vec{\jmath} - 2\vec{\kappa}$ and passing through the point $(1, 2, -1)$ and the plane $\mathcal{P}$ which is perpendicular on the vector $\vec{u} = -\vec{\imath} - \vec{\jmath} + 2\vec{\kappa}$ and passes through the point $(3, 1, 0)$. Answer the following:

a) Compute the intersection $L \cap \mathcal{P}$.

b) Does L intersect the sphere of radius 2 and center $(3, 2, 1)$?

3. Compute the point on the intersection line of the planes $x - y - z = 2$ and $x + y - z + 2 = 0$ which is closest to the point $(1, 1, 1)$.

4. Which plane contains the diagonal in the xy-plane and is perpendicular on the plane $x - y + z = 1$?

5. Find the distance from the z-axis to the intersection line of the planes $x = y + z + 1$ and $z = 2x + 2y + 1$.

6. Find the value of the x-coordinate where the plane through the points $(4, 1, 1)$, $(1, 2, 1)$, and $(1, 1, 2)$ intersects the x-axis.

7) Describe the set of all the points in the xy-plane which have equal distance to:

a) the points $(2, 3)$ and $(1, 1)$.

b) the lines $x + y = 2$ and $2x - y = 0$.

8) Describe the set of all the points in the xyx-space which have equal distance to:

a) the points $(2, 3, 0)$ and $(0, 1, 1)$.

b) the line $\vec{r}(t) = (1, 1, 1) + t(-1, 0, 2)$, $t \in \mathbb{R}$ and the plane $2x - y - z = 1$.

c) the planes $x + y + z = 1$ and $x - y - 2z = 1$.

9 Answer whether the following is true or false, and give a **reason / counterexample**: Let $\vec{u}$ and $\vec{v}$ be non-zero vectors in the xyz-space.

a) If $\vec{u} - a\vec{v}$ and $\vec{u} + a\vec{v}$ have the same length for some number $a \neq 0$, then $\vec{u} \perp \vec{v}$.

b) If $|\vec{u} + a\vec{v}| = |\vec{u}| + |a||\vec{v}|$ for some $a \neq 0$, then $a > 0$ and $\vec{u} = a\vec{v}$.

[**Hint:** What is the length of a vector in terms of dot product?]

10. Answer whether the following is true or false, and give a **reason / counterexample**: Let $\vec{u}, \vec{v}, \vec{w}$ be unit vectors in the xyz-space such that $\vec{u} \perp \vec{v}$, $\vec{v} \perp \vec{w}$ and $\vec{w} \perp \vec{u}$. Then the vector $\big(\vec{v} \times (\vec{w} \times \vec{u})\big) \times \vec{u}$ is a unit vector, i.e., has magnitude one. **Justify your answer!**

11. Let $\vec{u}_0$ be the unit with positive $\vec{\imath}$ and $\vec{\jmath}$ components and parallel to $-3\vec{\imath} - \vec{\jmath} - \vec{\kappa}$, and $\vec{v}_0$ be the unit vector having positive $\vec{\kappa}$ component and parallel to the intersection line of $x + y = 3z$ and $y + 2z = x$. Compute the volume of the parallelepiped spanned by $\vec{u}_0, \vec{v}_0, \vec{\imath} + \vec{\kappa}$.

[**Hint:** What is the volume of the parallelepiped spanned by three vectors?]

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