Due: Friday, September 28, 2018

Math 114 / Homework # 3 (one page)

NOTE: The core problems below are from Chapters 13, 14 of:
Calculus: Early Transcendentals, 2[nd] Custom Edition for UPenn, by Thomas et al..

- Before getting started, **read/study** Sections 12.1-5, 13.1-3, 14.1 and make sure that you understand the examples from the relevant sections !
- Work out the **Core Problems** (not necessary to submit them):

  13.2 Integrals of Vector Functions: 1, 13, 18, 21, 30, 33, 37.

  13.3 Arc Length in Space: 5, 12, 17, 19.

  14.1 Functions of Several Variables: 3, 9, 14, 18, 31, 32, 33, 34, 35, 36, 39, 50, 55, 62, 65.
- **Work out and submit** the suggested additional Problems:

1) Find the distance from the origin to the tangent line at $t = 1$ to the space curve defined by $\vec{r}(t)$, provided $\vec{r}(t) = \big(x(t), y(t), z(t)\big)$ satisfies:

$$\frac{d^2\,\vec{r}}{dt^2} = (0, -12t^2 + 2, e^{-t}), \qquad \frac{d\,\vec{r}}{dt}(1) = (\tfrac{1}{2}, 0, 0), \qquad \vec{r}(0) = (1, -1, 0).$$

2) Find the point P on the intersection line of the planes $x+y-2z = 0$ and $x+y-z+2 = 0$ which is closest to $Q = (1, 0, 1)$. Does P lie in the sphere of radius 1 centered at $(1, 1, 1)$?

3) Find all the the planes which contain the diagonal line in the yz-plane and are perpendicular on the plane $x - y + z = 1$.

4) Find the distance from the diagonal in the yz-plane to the intersection line of the planes $x = y + z + 1$ and $z = 2x + 2y + 1$.

5) Compute the angle (in radians) of the tangent line to the space curve defined by $\vec{r}(t) = e^t\,\vec{\imath} + (\cos^2 t + 1)\,\vec{\jmath} + (-t^2 + t - 1)\,\vec{\kappa}$ at the point $P_0 = \vec{r}(0)$ and the yz-plane.

6) Consider the unit vector $\vec{u}_1$ parallel to $-2\vec{\imath} - \vec{\jmath} - 2\,\vec{k}$ and having positive $\vec{\jmath}$ component, and the unit vector $\vec{u}_2$ having non-negative $\vec{k}$ component and parallel to the intersection line of $x + y - 3z = 0$ and $y + 2z = x$. What is the volume of the parallelepiped spanned by $\vec{u}_1$, $\vec{u}_2$, $\vec{\imath} + \vec{k}$. [**Hint:** What is the volume of the parallelepiped spanned by three vectors?...]

7) Find the real values $a, b, c \geqslant 0$ such that the space curves defined by the following vector-valued functions $\vec{s}(t) = \big(t + a, t, \sin^2 t\big)$ and $\vec{r}(t) = \big(e^t - b, \sin(t), \ln(x^2 + c)\big)$ pass through the origin of the xyz-space at $t = 0$. Is it true that the curves are orthogonal to each other at the origin of the xyz-sapce ? **Justify your answer!**

[**Hint:** What does mean "the curves are orthogonal at a point" in terns of their tangent lines at the point?...]

8) Find the coordinates of the point where the plane through the points $(4, 1, 2)$, $(1, 2, -1)$, and $(1, 1, 1)$ intersects the diagonal in the xz-plane.

9) Consider the space curve $\vec{r}(t) = (2t + a, b\ln t, ct^2)$. Knowing that the tangent line to the curve for $t = 1$ has parametric equation $(x, y, z) = (2, 0, 1) + s\,\vec{v}$, $-\infty < s < \infty$ and $\vec{v} = \vec{\imath} + \frac{1}{2}\vec{\jmath} + \vec{\kappa}$, compute the curve arc length for $1 \leqslant t \leqslant e$.