Due: Monday, October 8, 2018

Math 114 / Homework # 4 (one page)

NOTE: All the problems below are from Chapter 14 of:
Calculus: Early Transcendentals, 2nd Custom Edition for UPenn, by Thomas et al..

- Before getting started, **read/study Sections 14.2, 14.3, 14.4** and make sure that you understand the relevant definitions and examples (and know the old Sections 12.1-5, 13.1-3, 14.1) !
- Work out the **Core Problems** (not necessary to submit them):
  - 14.2 Limits and Continuity: 1, 9, 16, 27, 32, 41, 49, 56, 61
  - 14.3 Partial Derivatives: 5, 22, 26, 39, 46, 54, 63, 65, 73, 83, 90.
  - 14.4 The Chain Rule: 3, 7, 12, 14, 25, 31, 35, 41, 45, 50, 51.
- **Work out and submit** the suggested additional Problems:

1) Consider the vectors $\vec{v}_1 = \langle 6, a, 0\rangle$, $\vec{v}_2 = \langle 3, 4, -a\rangle$, $\vec{v}_3 = \langle 1, 2, 2\rangle$. Find all the parallelepipeds of maximal volume spanned by the three vectors. **Justify your answer** !

[**Hint**. Such parallelepipeds of maximal volume might not exist (WHY).]

2) Consider all the vectors $\vec{v} = a\,\vec{\imath} + b\,\vec{\jmath} + c\,\vec{\kappa}$ which are perpendicular on both $\langle 1, 0, 1\rangle$ and $\langle 1, 2, -1\rangle$, and satisfying: The parallelogram spanned by $\vec{v}$ and $\langle 0, 1, -1\rangle$ equals $5\sqrt{6}$. How many such vectors $\vec{v}$ exist, and what is their magnitude?

3) Find all numbers a such that the tangent vector $\vec{v}(t)$ to $\vec{r}(t) = \big(t\sin(2t), t\cos(2t), at^3\big)$ satisfies $\lim\limits_{t\to\infty} |\vec{v}(t)|/t^2 = 2$, and for each a find t_0 such that arc length for $0 \leqslant t \leqslant t_0$ is 4.

4) For the function $f(x,y) = \begin{cases} 4xe^{x^2y-5y^3}\sin(1/x) & \text{if } x \neq 0 \\ e^{-5y^3} & \text{if } x = 0 \end{cases}$ and answer the following:

a) Compute the limit $\lim\limits_{(x,y)\to(0,0)} f(x,y)$, if the limit exists.

b) Is $f(x,y)$ continuous on domain of definition?

5) For the functions below, find the value of $a, b \in \mathbb{R}$, **if such values exist**, s.t. the functions below are continuous on their domains of definition:

a) $f(x,y) = \begin{cases} \frac{\ln(3x^2-x^2y^2+3y^3)}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ a & \text{if } (x,y) = (0,0) \end{cases}$ b) $f(x,y,z) = \begin{cases} \frac{\tan(xyz)}{xyz} & \text{if } xyz \neq 0 \\ b & \text{if } xyz = 0 \end{cases}$

6) Solve the problems 64, 66, 74, 84, 92 from Section 14.3

7) Solve problems 36, 47, 52 from Section 14.4