Math 114 - Practice Final Exam - University of Pennsylvania

• Name: 

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My signature below certifies that I have complied with the University of Pennsylvania’s Code of Academic Integrity in completing this exam. I certify that all of the work on this test is my own.

Signature: ________________________________

INSTRUCTIONS:

1. Check your exam to make sure all 17 pages are present.

2. You may write on the back of pages, and also use the blank pages at the end of the exam for additional space. Indicate clearly where your work can be found.

3. Complete the information requested above.

4. You may use writing implements and a single handwritten sheet of 8.5”x11” paper.

5. No calculators, electronic devices, books or other aids are allowed.

6. Show all your work on the exam itself. Correct answers with little or no supporting work will not be given credit. We reserve the right to take off points if we cannot see how you arrived at your answer (even if your final answer is correct).

7. Good luck!

OFFICIAL USE ONLY:

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Your total score: _____________________
1. If \( \mathbf{a} + \mathbf{b} \) is parallel to \( \mathbf{a} - \mathbf{c} \), \( \mathbf{a} \times \mathbf{b} = \langle 1, 0, 1 \rangle \), and \( \mathbf{a} \times \mathbf{c} = \langle 1, 1, 0 \rangle \), what is \( \mathbf{b} \times \mathbf{c} \)?

(a) \( \langle 1, 0, 1 \rangle \)  
(b) \( \langle 1, 2, 1 \rangle \)  
(c) \( \langle 2, 1, 1 \rangle \)  
(d) \( \langle 0, 0, 0 \rangle \)  
(e) \( \langle -1, 0, -1 \rangle \)  
(f) \( \langle -1, -2, -1 \rangle \)  
(g) \( \langle -2, -1, -1 \rangle \)  
(h) None of the above

Answer: __________
2. Consider the line $L_1$ through the points $(1, 1, 0)$ and $(2, 3, 0)$, and the line $L_2$ through the points $(0, 0, 0)$ and $(1, -1, 1)$. What is the distance between these lines (i.e. the shortest possible distance between a point from $L_1$ and a point from $L_2$)? (Hint: the line connecting two points at the shortest distance will be perpendicular to both $L_1$ and $L_2$.)

(a) $1/4$  (b) $1/2$  (c) $3/4$  (d) $1/\sqrt{2}$
(e) $\sqrt{3}/4$  (f) $1/\sqrt{5}$  (g) $\sqrt{5}/4$  (h) None of the above

Answer: ________
3. A particle travels along the curve \( \mathbf{r}(t) = (5 \cos t, 5 \sin t, 12t) \). If a particle starts at \((5, 0, 0)\) and moves along the curve (in the direction of increasing \(t\)), at what point is the total distance moved by the particle equal to \(\frac{13\pi}{2}\)?

Answer: _________
4. Find the constant $k$ that makes the function

\[ f(x, y) = \begin{cases} 
\sqrt{x^{10} + x^5 y^5}, & (x, y) \neq (0, 0) \\
\frac{x^5 + y^5}{k}, & (x, y) = (0, 0)
\end{cases} \]

continuous at $(0, 0)$.

(a) $-2$  (b) $\frac{2}{3}$  (c) $0$  (d) $-\frac{1}{2}$
(e) $1$  (f) $-\frac{3}{2}$  (g) No such value of $k$ exists  (h) None of the above

Answer: __________
5. Find the directional derivative of the function

\[ f(x, y, z) = \ln (x^2 + y^2 + z^2) + e^{x+y} \]

at \((2, -2, 2)\) in the direction in which \(f\) increases most rapidly.

(a) \(\sqrt{\frac{11}{3}}\)  
(b) \(\sqrt{\frac{7}{2}}\)  
(c) \(\sqrt{\frac{7}{3}}\)  
(d) \(\sqrt{\frac{11}{2}}\)

(e) \(4\)  
(f) \(\sqrt{\frac{11}{4}}\)  
(g) \(\sqrt{\frac{86}{5}}\)  
(h) None of the above

Answer: _________
6. Find the minimum and maximum values of $xy + z$ on the sphere $x^2 + y^2 + z^2 = 1$.

(a) maximum: $1 + \sqrt{3}$, minimum: $1 - \sqrt{3}$  
(c) maximum: $1$, minimum $-\sqrt{3}$  
(e) maximum: $2$, minimum $0$  
(g) maximum: $2$, minimum $-1$

(b) maximum: $2$, minimum: $\sqrt{3}$  
(d) maximum: $\sqrt{2}$, minimum $1$  
(f) maximum: $1$, minimum $-1$  
(h) None of the above

Answer: __________
7. Find the average value of the function \( f(x, y) = x + y \) on the region \( 0 \leq x \leq y \leq 1 \).

(a) 1  
(b) 2  
(c) 3  
(d) \( \frac{1}{2} \)

(e) \( \frac{2}{3} \)  
(f) \( \frac{1}{3} \)  
(g) \( \frac{1}{6} \)  
(h) None of the Above

Answer: _________
8. Find the volume of the ice-cream cone enclosed by $x^2 + y^2 + z^2 = 4$ and $z = \sqrt{x^2 + y^2}$.

(a) $\frac{2 - \sqrt{2}}{3} \pi$
(b) $\frac{2\pi}{3}$
(c) $\frac{\pi^2}{2}$
(d) $\frac{16 - 8\sqrt{2}}{3} \pi$

(e) $\frac{16\pi}{3}$
(f) $\frac{\pi}{3}$
(g) $\frac{8\pi}{3}$
(h) $\frac{17\pi}{6}$

Answer: _________
9. Find the iterated integral

\[ \int_{-2\pi}^{2\pi} \int_{\frac{|x|}{2}}^{\pi} \frac{\sin y}{y} \, dy \, dx \]

(a) 4  (b) 2  (c) 2\pi  (d) 4\pi
(e) 8  (f) 0  (g) 16  (h) \frac{1}{2}

Answer: _________
10. Find the work (equivalently, the flow) of the vector field \( \vec{F} = x\vec{i} + y\vec{j} + z\vec{k} \) along the curve \( C \) defined by \( \vec{r}(t) = t^2\vec{i} + (t \sin t)\vec{j} + (t \cos t)\vec{k} \), where \( 0 \leq t \leq 1 \).

(a) \(-\pi/2\)  (b) \(-1\)  (c) \(-\pi/3\)  (d) 0 
(e) \(\pi/3\)  (f) 1  (g) \(\pi/2\)  (h) None of the Above

Answer: [blank]
11. \( a, b, \) and \( c \) are constants so that

\[
\vec{V} = (x + ay + z)\vec{i} + (y + bz)\vec{j} + (cx - y)\vec{k}
\]

is a conservative vector field. What are \( a, b, c \) and what is a function \( f \) so that \( \nabla f = \vec{V} \)?

Answer: __________
12. Find the area of the part of the plane $3x + 2y + z = 6$ that lies in the first octant.

(a) $5\sqrt{11}$  
(b) $7\sqrt{13}$  
(c) $7\sqrt{14}$  
(d) $3\sqrt{13}$  
(e) $3\sqrt{14}$  
(f) $5\sqrt{13}$  
(g) $5\sqrt{14}$  
(h) None of the Above

Answer: ________
13. Evaluate the flux integral of $\nabla \times \mathbf{F}$ across the surface $S$ where

$$ \mathbf{F}(x, y, z) = x^2 z^2 \mathbf{i} + y^2 z^2 \mathbf{j} + xyz \mathbf{k}, $$

and $S$ is the part of the paraboloid $z = x^2 + y^2$ that lies inside the cylinder $x^2 + y^2 = 4$ oriented upward.

(a) 2 (b) 1 (c) 0 (d) $-1$ (e) $-2$ (f) $-3$ (g) $-4$ (h) None of the Above

Answer: 

[Diagram of a paraboloid intersecting a cylinder]
14. Calculate the flux of $\mathbf{F}$ across the surface $S$ where

$$\mathbf{F}(x, y, z) = 3xy^2 \mathbf{i} + xe^z \mathbf{j} + z^3 \mathbf{k}$$

and $S$ is the surface of the solid bounded by the cylinder $y^2 + z^2 = 1$ and the planes $x = -1$ and $x = 2$.

(a) $\frac{5}{2} \pi$ (b) $5\pi$ (c) $\frac{7}{2} \pi$ (d) $7\pi$ (e) $\frac{9}{2} \pi$ (f) $9\pi$ (g) $9\pi$ (h) None of the Above

Answer: __________
Extra space for work 1:
Extra space for work 2: