1) Let $k$ be an algebraically closed field, and $K = k((t))$ the Laurent power series field over $K$.

a) Suppose that $\text{char}(k) = 0$. Show that for every $n > 0$, $K$ has up to isomorphism exactly one extension $K_n / K$ of degree $n$.

b) Is the same true if $\text{char}(k) = p > 0$?

**Notation:** In the sequel, if not explicitly otherwise stated, $K$ denotes a complete field w.r.t. the discrete valuation $v$, having a perfect residue field $\kappa_K$. If $L / K$ is a finite extension, we denote by $R_L / R_K$ the corresponding extension of (complete) DVR’s, and by $\mathcal{D}_{L/K}$ the different of $R_L / R_K$.

2) Let $L / K$ denote arbitrary finite abelian extensions of $K$.

a) Show that in general there exist $L / K$ which cannot be represented as a compositum $L = L_0 L_1 L_2$ with $L_0 | K$ unramified, $L_1 | K$ totally tamely ramified, and $L_2 | K$ totally wildly ramified.

b) Try to figure out the most general conditions on $K$ such that the above decomposition is always possible.

c) Suppose that some $L | K$ can be represented in the form $K = L_0 L'$ with $L_0 | K$ unramified and $L' | K$ totally ramified. Prove or disprove: If $L_1 | K$ and $L_2 | K$ are maximal totally tamely, respectively totally wildly, ramified sub-extensions of $L | K$, then $L = L_0 L_1 L_2$.

3) Complete the details of the proof of **Hilbert’s Formula**:

Let $L | K$ some finite Galois extension, and $G_0 \supseteq \ldots G_i \supseteq \ldots \{1\}$ be the higher ramification groups of $\text{Gal}(L | K)$. Then one has:

$$v_L(\mathcal{D}_{L/K}) = \sum_{i \geq 0} (|G_i| - 1)$$

4) Let $L | K$ be a finite field extension. Prove or disprove:

a) $L | K$ is tamely ramified iff $e(L | K) - 1 = v_L(\mathcal{D}_{L/K})$.

b) $L | K$ is wildly ramified iff $e(L | K) - 1 < v_L(\mathcal{D}_{L/K})$.

5) Prove or disprove: If the residue field $\kappa_K$ is not perfect, then the assertions from Problems 1, 2 and 3 above do not necessarily hold.

6) Let $G$ be a finite $p$-group. Prove or disprove:

a) $G$ has a unique maximal $p$-elementary abelian quotient $\overline{G}$.

b) The minimal number of generators of $G$ equals the minimal number of generators of $\overline{G}$.

c) The above assertions hold in the same form for all pro-$p$ groups $G$.

7) Prove the Local Kronecker–Weber Theorem for $\mathbb{Q}_2$. 