Consider the usual context: \( k, K|k, k[X], k^n, \) etc.

**Basics**

1) Answer the following:
   a) Describe the affine \( k \)-algebraic subsets of \( \mathbb{A}^1 \) which are Zariski closed; when is such a subset irreducible, resp. absolutely irreducible?
   b) The same question for the 0-dimensional closed subsets of \( \mathbb{A}^n \).
   c) Is the answer similar, if we drop the hypothesis "0-dimensional" in case b)?

2) For \( n \geq 3 \), answer the following questions:
   a) What are the irreducible components of \( V_1 = V(2X_1^2 - 3X_2X_3), V_2 = V(X_1X_2 - X_1), \) and \( V_1 \cap V_2? \)
   b) What are the connected components of the above affine \( k \)-algebraic sets?

3) For \( n = 3 \), consider \( X = \{ (t, t^2, t^3) \mid t \in K \} \subset \mathbb{A}^3. \) Show that \( X \) is an absolutely irreducible affine \( k \)-algebraic set of dimension 1. (\( X \) is called the twisted cubic curve in \( \mathbb{A}^3 \).)

**Zariski closure**

For \( V \) an affine \( k \)-algebraic set, and \( T \subset V \), let \( \overline{T} \) denote the closure of \( T \) in \( V \) in the Zariski topology (for short, the Zariski closure of \( T \) in \( V \)). Further set \( I(T) = \{ f \in k[X] \mid f|_T = 0 \} \).

4) Show that \( V(I(T)) \) is the Zariski closure of \( T \) in \( \mathbb{A}^n \). What is the corresponding assertion, if we replace \( \mathbb{A}^n \) by \( V \)?

5) For a one element set \( \{ x \} \subset \mathbb{A}^n \), let \( V_x \) be the Zariski closure of \( x \) in \( \mathbb{A}^n \).
   a) Show that \( V_x \) is always irreducible. Prove or disprove: \( V_x \) is absolutely irreducible.
   b) Prove or disprove: When \( x \) varies, \( \dim(V_x) \) can be any number between 0 and \( n \).
   c) How can one deduce the coordinate ring \( k[V_x] \) of the affine \( k \)-alg. set \( V_x \) from \( x = (x_1, \ldots, x_n) \)?

**Definition.** In the context above, \( x \in V \) is called a generic point in classical sense of \( V \), if \( V = V_x \).

6) Given an affine \( k \)-algebraic subset \( V \subset \mathbb{A}^n \), prove the following:
   a) If \( V \) has a generic point, then \( V \) is irreducible.
   b) The converse of a) is true iff \( \text{tr.deg}(K|k) \geq \dim(V) \).
   c) Prove or disprove: The set of generic points in classical sense of \( V \) is either empty or infinite.

**Connectivity.**

7) Prove or disprove:
   a) \( U_1 = \mathbb{A}^1 \setminus \{0\} \) is connected as a subset of \( \mathbb{A}^1 \).
   b) The same question about \( U_2 = \mathbb{A}^n \setminus V, \) with \( V \) some affine \( k \)-algebraic subset of \( \mathbb{A}^n \).

Given an affine \( k \)-algebraic subset \( V \subset \mathbb{A}^n \), let \( k[V] \) be its coordinate ring, and \( kV \) denote the algebraic closure of \( k \) in \( k(V) \) the total fraction field of \( k[V] \).

8) Prove the following assertion made in the Lecture: Let \( V_\beta \subseteq V \) be a non-empty subset. Then \( V_\beta \) is a connected component of \( V \) iff there exists a minimal idempotent \( \pi \in k[V] \) such that \( V_\beta = V(1 - \pi) \).

9) Show that \( V \) is geometrically connected iff \( kV|k \) is a purely inseparable field extension.