Homework 4B

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MATH 241-910 :Calculus IV

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Problem 0. Reading: section 4.2-4.4 in Haberman. *Optional reading:* section 4.5 in Haberman

Problem 1. Using the separation of variables, solve the wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial t^2} \tag{1}$$

for $0 < x < L$ subjected to the the boundary conditions

$$u(0,t) = 0, \quad u(L,t) = 0, \quad u(x,0) = 2\sin\frac{\pi x}{L} + \frac{3\pi x}{L}, \quad \frac{\partial u}{\partial t}(x,0) = 3\sin\frac{4\pi x}{L}. \tag{2}$$

Problem 2. (*Damped vibrating string*)
Haberman 4.4.3

Problem 3. (*d'Alambert solution*) Haberman 4.4.7

Problem 4. (*Conservation of energy*) Define the energy of a vibrating string in the interval $0 < x < L$ to be

$$E(t) = \int_0^L \frac{1}{2}\left(\frac{\partial^2 u}{\partial t^2}\right)^2 + \frac{c^2}{2}\left(\frac{\partial^2 u}{\partial t^2}\right)^2 dx, \tag{3}$$

where c^2 is the constant in the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial t^2}$. Here $\int_0^L \frac{1}{2}(\frac{\partial^2 u}{\partial t^2})^2 dx$ is called the **kinetic energy**, and $\int_0^L \frac{c^2}{2}(\frac{\partial^2 u}{\partial x^2})^2 dx$ is called the **potential energy**.

1. Derive the *conservation of energy* formula

$$\frac{dE}{dt} = c^2 \frac{\partial u}{\partial x}\frac{\partial u}{\partial t}\bigg|_0^L. \tag{4}$$

2. How does E(t) change over time if $\frac{\partial u}{\partial x}(0,t) = 0$, $\frac{\partial u}{\partial x}(L,t) = 0$?

3. How does E(t) change over time if $u(0,t) = 0$, $u(L,t) = 0$?

Optional exercise: Below are exercise proposed in class and some additional problems. These are for your own benefit and may be helpful for your understanding of the material It is not required to turn them in. It is suggested that you at least read those problems.

Problem A. (*d'Alambert solution continues*) Haberman 4.4.8

Problem B. (*non uniform string*) Haberman 4.4.2 (We might do it in class)

Due: June 5 (World environment day), 2017, in class