Math 241-910 Calculus IV **Name:** _____
Summer 2017
Quiz 1
5/30/2017
Time Limit: 60 Minutes _____

This exam contains 6 pages (including this cover page) and 5 questions.
Total of points is 65.

Grade Table (for teacher use only)

Question	Points	Score
1	15	
2	15	
3	15	
4	15	
5	5	
Total:	65	

1. (15 points) Consider the heat equation

$$\frac{\partial u}{\partial t} = 5\frac{\partial^2 u}{\partial x^2} + 3x \tag{1}$$

in a rod $0 < x < \pi$. The initial condition is $u(x, 0) = \sin 3x$, and the boundary condition is $\frac{\partial u}{\partial x}(0, t) = b$, $\frac{\partial u}{\partial x}(\pi, t) = e^{-t}$. Denote the total thermal energy in the rod by

$$E(t) = \int_0^\pi u(x, t)dx. \tag{2}$$

   1. Compute $\frac{dE}{dt}$. (*Hint: you do <u>not</u> have to solve for $u(x, t)$ first*).
   2. Using part (a), find the total thermal energy $E(t)$.
   3. For which value of b does the limit $\lim_{t\to+\infty} E(t)$ exist? Compute the limit if it does exist.

2. (15 points) Consider the eigenvalue problem

$$\frac{d^2\phi}{dx^2} + \lambda\phi = 0. \tag{3}$$

Determine the eigenvalues λ (and corresponding eigenfunctions) if ϕ satisfies the boundary conditions

$$\frac{d\phi}{dx}(0) = 0 \quad \phi(L) = 0. \tag{4}$$

Analyze three cases ($\lambda > 0, \lambda = 0, \lambda < 0$). You may assume the eigenvalues are real.

3. (15 points) Consider the heat equation

$$\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial x^2} \tag{5}$$

for $0 \leq x \leq 3$, $t \geq 0$ with the following initial/boundary conditions

$$\frac{\partial u}{\partial x}(0,t) = 0, \quad \frac{\partial u}{\partial x}(3,t) = 0 \tag{6}$$

$$u(x,0) = 3 - \cos(3\pi x). \tag{7}$$

Solve the equation. What is $u(\frac{1}{2}, 2)$?

4. (15 points) Solve the Laplace equation

$$\Delta u = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0 \tag{8}$$

on the dish of radius 1 which is bounded at the origin, i.e. $|u(0,\theta)| < +\infty$, and satisfies the boundary condition $u(1,\theta) = 7 + 9\cos\theta$.

5. (5 points) (extra credits)

   Do you have any feedback, recommendations, suggestions for either the course or the teaching?