Math 241-910 Calculus IV **Name:** ____________________
Summer 2017
Quiz 2
6/6/2017
Time Limit: 60 Minutes ____________________

This exam contains 6 pages (including this cover page) and 5 questions.

Total of points is 70.

Grade Table (for teacher use only)

Question	Points	Score
1	15	
2	15	
3	15	
4	15	
5	10	
Total:	70	

1. (15 points) Consider the function

$$f(x) = \begin{cases} 2x, & 0 \le x \le \frac{1}{2} \\ 1, & \frac{1}{2} \le x \le 1 \end{cases} \tag{1}$$

   1. Compute the Fourier sine series of $f(x)$.
   2. Sketch the Fourier sine series.
   3. At which points of $[0, 1]$ does the Fourier sine series of $f(x)$ converges to $f(x)$?

2. (15 points) For what values of a and b is the function

$$f(x) = x^3 + ax + b \tag{2}$$

orthogonal to both $f_1(x) = 1$ and $f_2(x) = x$ on the interval $[-1, 1]$?

3. (15 points) Define the energy of a vibrating string in the interval $0 < x < L$ to be

$$E(t) = \int_0^L \frac{1}{2}\left(\frac{\partial u}{\partial t}\right)^2 + \frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2 dx, \tag{3}$$

corresponding to the wave equation $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, 0 < x < 1, t > 0$. Suppose $\frac{\partial u}{\partial x}(0,t) = 0 = \frac{\partial u}{\partial x}(1,t)$, show that the total energy is conserved, i.e. $E(t) = E(0)$ for all $t > 0$.

4. (15 points) Solve the Initial/Boundary value problem of wave equations

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} - 5u, \quad 0 < x < \pi, t > 0 \tag{4}$$

$$u(0,t) = u(L,t) = 0 \tag{5}$$

$$u(x,0) = 3\sin(2x) \tag{6}$$

$$\frac{\partial u}{\partial t}(x,0) = 4\sin(6x) \tag{7}$$

5. (10 points) (extra credits)

   Here's an interesting puzzle:

   There is an island upon which a tribe resides. The tribe consists of 1000 people, with various eye colours. Yet, their religion forbids them to know their own eye color, or even to discuss the topic; thus, each resident can (and does) see the eye colors of all other residents, but has no way of discovering his or her own (there are no reflective surfaces). If a tribesperson does discover his or her own eye color, then their religion compels them to commit ritual suicide at noon the following day in the village square for all to witness. All the tribespeople are highly logical and devout, and they all know that each other is also highly logical and devout (and they all know that they all know that each other is highly logical and devout, and so forth).

   Of the 1000 islanders, it turns out that 100 of them have blue eyes and 900 of them have brown eyes, although the islanders are not initially aware of these statistics (each of them can of course only see 999 of the 1000 tribespeople).

   One day, a blue-eyed foreigner visits to the island and wins the complete trust of the tribe.

   One evening, he addresses the entire tribe to thank them for their hospitality.

   However, not knowing the customs, the foreigner makes the mistake of mentioning eye color in his address, remarking "how unusual it is to see another blue-eyed person like myself in this region of the world".

   What effect, if anything, does this faux pas have on the tribe?

The interesting thing about this puzzle is that there are two quite plausible arguments here, which give **opposing** conclusions:

Argument 1. The foreigner has no effect, because his comments do not tell the tribe anything that they do not already know (everyone in the tribe can already see that there are several blue-eyed people in their tribe).

Argument 2. 100 days after the address, all the blue eyed people commit suicide. This is proven as a special case of

Suppose that the tribe had n blue-eyed people for some positive integer n. Then n days after the traveller's address, all n blue-eyed people commit suicide.

Which argument do you think is correct? Write down what you think. Try to prove the proposition in argument 2.