Homework Set 8

DUE: NOV 27 - 29, 2017 (AT THE BEGINNING OF RECITATION)

1. Find the Taylor Series centered at $x_0 = 0$ for the following functions:
   (a) $f(x) = x^2 e^{3x}$
   (b) $f(x) = x\cos(x^2)$
   (c) $f(x) = \ln(1 + 2x + x^2)$

2. Use the first 3 terms in the Taylor Series (that is, the Taylor Polynomial of degree 3) of $f(x) = x\cos(x^2)$ to approximate the value of $\cos(1/4)$.

3. A tank with brine has pure water flowing into it at a rate of 12 ℓ/min. The contents of the tank are kept thoroughly mixed, and the contents flow out at a rate of 10 ℓ/min. Initially, the tank contains 10 kg of salt dissolved in 100 ℓ of water. How much salt will be dissolved in the water after 30 min?

4. The *logistic model* provides a more realistic model to population growth, taking into account the finiteness of resources available. In this model, denoting by $y(t)$ the population at time t, the relative growth rate of the population $y'(t)/y(t)$ is postulated to decrease when $y(t)$ achieves the carrying capacity K of the environment. That is,

$$\frac{\mathrm{d}y}{\mathrm{d}t} = r\, y(t)\left(1 - \frac{y(t)}{K}\right).$$

   (a) Using that this first order equation is separable, find its general solution $y(t)$.
   (b) The population of the USA in the years 1900 and 1950 was estimated to be 76 million and 150 million, respectively. Using the *exponential growth model*, what is the predicted population of the USA in the year 2000 and in the year 2050? You may use a calculator to approximate values of natural logarithms.
   (ANSWER: Approx. 296 million in year 2000, and 584 million in year 2050.)
   (c) The actual population of the USA in the year 2000 census was estimated to be 281 million. With this information (and the population data of 1950), refine the above prediction for the year 2050 using the logistic model.
   (ANSWER: Approx. 483 million in year 2050, using $K \cong 1421.07$ and $r = -0.0147$.)