

Practice Problems for Midterm 1

1. What is the volume of the solid obtained by revolving the region bounded by the line $y = 2x$ and the parabola $y = 2 - x^2$ about the line $x = 1$?

2. Find the volume of the solid obtained by revolving the region bounded between the x -axis and the curve $y = \cosh x$, $-2 \leq x \leq 2$, about the x -axis.

HINT: Recall that $\cosh x = \frac{e^x + e^{-x}}{2}$.

3. Find the volume of the solid obtained by revolving about the x -axis the region bounded by the curve $y^2 = \cos x$ and the lines $y = 0$, $y = 1$, $x = -\frac{\pi}{2}$, and $x = \frac{\pi}{2}$.

4. Find the length of the curve $y = \frac{x^4}{4} + \frac{1}{8x^2}$ for $1 \leq x \leq \sqrt{2}$.

5. Find the length of the curve $y = \frac{e^x}{4} + \frac{1}{e^x}$ for $0 \leq x \leq 1$.

6. Use an integral to find the surface area of the (side of the) cone obtained by revolving the line $y = 2x$ about the x -axis, for $0 \leq x \leq 3$.

7. Use an integral to find the surface area of the (side of the) cone obtained by revolving the line $y = 2x$ about the y -axis, for $0 \leq x \leq 3$.

8. What is the centroid of the region bounded by the curves $y = e^x$ and $y = -1$ for $0 \leq x \leq \ln 2$?

9. Compute the following trigonometric integrals:

a) $\int_0^{\pi/2} \sin(x) \cos^3(x) \, dx$

b) $\int_0^{\pi/2} \sin^2(x) \cos^3(x) \, dx$

c) $\int_0^{3\pi/2} \sin^3(x) \cos^3(x) \, dx$

d) $\int_0^{\pi/6} \tan^4(x) \sec^4(x) \, dx$

10. Use integration by parts to compute the following indefinite integrals:

a) $\int x e^x dx$

b) $\int e^{-x} \sin(x) dx$

c) $\int x^2 \sin(4x) dx$

d) $\int \ln(4x) dx$

11. Use a trigonometric substitution to compute the following indefinite integrals:

a) $\int \frac{x}{\sqrt{9-x^2}} dx$

b) $\int \frac{1}{\sqrt{1+x^2}} dx$

c) $\int \sqrt{1-9x^2} dx$

d) $\int \frac{1}{\sqrt{4x-x^2}} dx$

12. Use partial fractions to evaluate the following definite integrals:

a) $\int_2^3 \frac{1}{x^2+2x-3} dx$

b) $\int_4^8 \frac{x}{x^2-2x-3} dx$

c) $\int_0^1 \frac{x^3}{1+2x+x^2} dx$

d) $\int_0^1 \frac{1}{(x^2+1)(x+1)} dx$