# HW2.

**Problem 1.**

$$\text{Length} = \int_1^4 \sqrt{1+(f'(x))^2}\,dx$$

$$= \int_1^4 \sqrt{1+\left(\frac{x^3}{4}-\frac{1}{x^3}\right)^2}\,dx$$

$$= \int_1^4 \sqrt{\left(\frac{x^3}{4}+\frac{1}{x^3}\right)^2}\,dx = \int_1^4 \left(\frac{x^3}{4}+\frac{1}{x^3}\right)dx$$

$$= \left(\frac{x^4}{16}-\frac{1}{2x^2}\right)\Big|_1^4 = \left(16-\frac{1}{32}\right)-\left(\frac{1}{16}-\frac{1}{2}\right)$$

$$= \frac{525}{32}$$

**Problem 2.**

$$\text{Arch length} = \int_{-2}^{2} \sqrt{1+[f'(x)]^2}\,dx$$

$$= \int_{-2}^{2} \sqrt{1+\left[\frac{2\sqrt{3}}{9}\cdot\frac{3}{2}6x\,(3x^2+1)^{\frac{1}{2}}\right]^2}$$

$$= \int_{-2}^{2} \sqrt{1+12x^2(3x^2+1)}\,dx$$

$$= \int_{-2}^{2} \sqrt{36x^4+12x^2+1}\,dx$$

$$= 2\int_0^2 (6x^2+1)\,dx = 2\,(2x^3+x)\Big|_0^2$$

$$= 2\cdot(16+2) = 36$$

3. Surface Area $= \displaystyle\int_{\frac{1}{2}}^{1} 2\pi f(x) \cdot \sqrt{1+[f'(x)]^2}\, dx$

$$= 2\pi \int_{\frac{1}{2}}^{1} \sqrt{x}\ \sqrt{1+\frac{1}{4x}}\ dx$$

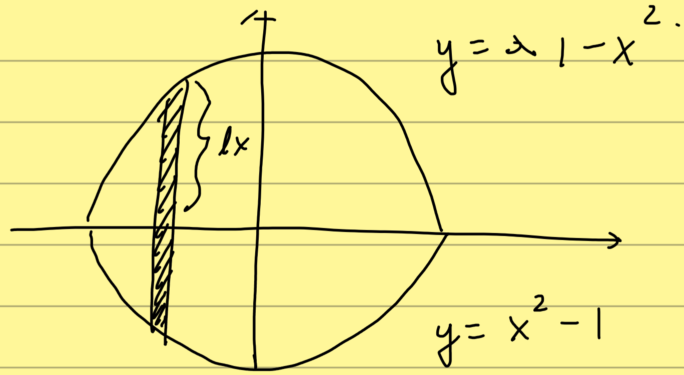
$$= 2\pi \int_{\frac{1}{2}}^{1} \sqrt{x+\frac{1}{4}}\ dx$$

$$= 2\pi\ \frac{2}{3}\left(x+\frac{1}{4}\right)^{\frac{3}{2}} \Big|_{\frac{1}{2}}^{1}$$

$$= \frac{4\pi}{3}\left[\left(\frac{5}{4}\right)^{\frac{3}{2}} - \left(\frac{3}{4}\right)^{\frac{3}{2}}\right]$$

$$= \frac{4\pi}{3}\left(\frac{5\sqrt{5}-3\sqrt{3}}{8}\right)$$

4.




$l_x = 2(1-x^2)$.

By symmetry, $M_y = 0$.

$$M_x = \frac{\int_{-1}^{1} x \cdot 2 l_x \cdot \delta(x,y) \cdot dx}{\int_{-1}^{1} 2 l_x \cdot \delta(x,y)\, dx}$$

$$= \frac{\int_{-1}^{1} x \cdot 2 \cdot 2(1-x^2) \cdot (-x^2+2x+3)\, dx}{\int_{-1}^{1} 2 \cdot 2(1-x^2) \cdot (-x^2+2x+3)\, dx}$$

By odd-even argument.

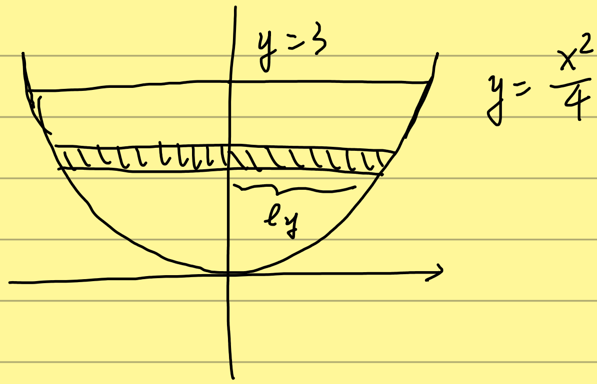
$$= \frac{\int_{-1}^{1} x(1-x^2) \cdot 2x\, dx}{\int_{-1}^{1} (1-x^2)(-x^2+3)\, dx}$$

$2x^2 - 2x^4$

$(3-4x^2+x^4)$

$$= \frac{\frac{2}{3} - \frac{2}{5}}{3 - \frac{4}{3} + \frac{1}{5}} = \frac{1}{7}$$

5.




$l_y = x = \sqrt{4y}$

By symmetry, $M_x = 0$

$l_y \cdot dy$ is the area of the shaded strip.

$$M_y = \frac{\int_0^3 y \cdot l_y \, dy}{\int_0^3 l_y \, dy} = \frac{\int_0^3 2y^{\frac{3}{2}} \, dy}{\int_0^3 2y^{\frac{1}{2}} \, dy}$$

$$= \frac{2 \cdot \frac{2}{5} y^{\frac{5}{2}} \Big|_0^3}{2 \cdot \frac{2}{3} \cdot y^{\frac{3}{2}} \Big|_0^3} = \frac{\frac{4}{5} \cdot 3^{\frac{5}{2}}}{\frac{4}{3} \cdot 3^{\frac{3}{2}}} = \frac{9}{5}$$