1. (a) $\int x^3 e^x dx$

$= x^3e^x - \int 3x^2 \cdot e^x dx$

$= x^3e^x - (3x^2e^x - \int 6x e^x dx)$

$= x^3e^x - [3x^2e^x - (6x \cdot e^x - \int 6e^x dx)]$

$= x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C$

Similarly, $\int x^2e^x = x^2e^x - 2xe^x + e^x + C$

So $\int (x^3 - 2x^2)e^x$

$= e^x(x^3 - 3x^2 + 6x - 6) - 2e^x(x^2 - 2x + 1) + C$

(b) $\int \cos 2z \cdot \cos 5z \, dz$

$= \int \frac{\cos 7z + \cos 3z}{2} dz$

$= \frac{1}{2}\left(\frac{\sin 7z}{7} + \frac{\sin 3z}{3}\right) + C$

(c) Recall that $\int x\cos^2 x dx = \int \frac{1 - \sin 2x}{2} dx$

$= \frac{x}{2} + \frac{\cos 2x}{4}$

Using integral by part we get

$x \cdot \left(\frac{x}{2} + \frac{\cos 2x}{4}\right) - \int 1 \cdot \left(\frac{x}{2} + \frac{\cos 2x}{4}\right) dx$

$$= x \cdot \left(\frac{x}{2} + \frac{\cos 4x}{2} \right) - \left(\frac{x^2}{4} + \frac{\sin 2x}{8} \right) + C$$

d) $\int x e^{3x} dx$

integral by part

$$= x \cdot \frac{e^{3x}}{3} - \int 1 \cdot \frac{e^{3x}}{3} dx$$

$$= \frac{x e^{3x}}{3} - \frac{e^{3x}}{9} + C$$

e). $\int \sec^3 x \, dx =$

Recall that $(\tan x)' = \sec^2 x$

$\sec^2 x = 1 + \tan^2 x$

$$= \int \sec x \cdot d\tan x = \int \sqrt{1 + \tan^2 x} \, d\tan x$$

$\underline{\underline{u = \tan x}}$ $\int \sqrt{1 + u^2} \, du$

$\underline{\underline{\text{By integral table}}}$ $\frac{u}{2} \sqrt{1 + u^2} + \frac{1}{2} \ln\left(u + \sqrt{1 + u^2} \right) + C$

$\underline{\underline{u = \tan x}}$ $\frac{\tan x}{2} \cdot \sec x + \frac{1}{2} \ln\left(\tan x + \sec x \right) + C$

integral by part.

(f) $\int e^{-2y} \sin y \, dy = e^{-2y}(-\cos y) - \int -2 e^{-2y}(-\cos y) dy$

$= -e^{-2y}\cos y - 2\left(e^{-2y}\sin y - \int -2e^{-2y}\sin y dy\right)$

i.e. $\int e^{-2y}\sin y\, dy = -e^{-2y}\cos y - 2e^{-2y}\sin y - \int 4 e^{-2y}\sin y\, dy$

solving $\int e^{-2y}\sin y dy$ in this equation

i.e. $\int e^{-2y}\sin y\, dy = \frac{1}{5}\left(-e^{-2y}\cos y - 2e^{-2y}\sin y\right) + C$

(g) Let $\sin x = u$. $du = \cos x dx$

$\int \sin^3 x \cos^3 x dx = \int u^3(1-u^2)\, du$

$= \int (u^3 - u^5)\, du = \frac{u^4}{4} - \frac{u^6}{6} + C$

$= \frac{\sin^4 x}{4} - \frac{\sin^6 x}{6} + C$

h) Let $\sqrt{x} = u$. $du = \frac{1}{2} x^{-\frac{1}{2}} dx$

$$\int \sqrt{x}\, e^{\sqrt{x}} dx = \int u \cdot e^{u}\, 2u\, du$$

$$= 2\int u^2 e^{u} du$$

By (a)

$$= 2\left(u^2 e^{u} - 2u \cdot e^{u} + 2 \cdot e^{u} \right) + C$$

i) Let $u = \sin\theta$. then $du = \cos\theta d\theta$

$$\int \sin^2\theta \cos\theta d\theta = \int u^2 du = \frac{u^3}{3} + C$$

$$= \frac{\sin^3\theta}{3} + C$$

j) Recall that $\cos 2w = 1 - 2\sin^2 w$

$$\int \sqrt{1 - \cos 2w}\, dw = \int \sqrt{1 - (1 - 2\sin^2 w)}\, dw$$

$$= \int \sqrt{2} \sin w\, dw$$

$$= -\sqrt{2} \cos w + C$$

k) Let $u = \ln x$. then $du = \frac{1}{x} \cdot dx$.

$$\int \frac{1}{x \cdot \ln x} dx = \int \frac{du}{u} = \ln u + C$$

$$= \ln(\ln x) + C$$

$$\int \frac{1}{x(\ln x)^2}\,dx \xlongequal{\text{let } u=\ln x} \int \frac{du}{u^2}$$

$$= -u^{-1}+C = \frac{-1}{\ln x}+C$$