Hw 8

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(a) $\sum (-1)^n \frac{1}{n}$. By alternating sum test it converges since $\frac{1}{n} \to 0$, decreasing.

But $\sum \frac{1}{n}$ diverges. so it converges conditionally.

(b) Same reason as above. converges conditionally

(c) $\sum_{n=1}^{\infty} (-1)^n \frac{e^n}{n^2}$ Notice that $\lim_{n\to+\infty} \frac{e^n}{n^2} = +\infty$,

thus $\lim a_n \neq 0$, diverges.

(d) the absolute part $\sum \frac{1}{n^2+1}$ converges by comparing to $\sum \frac{1}{n^2}$. i.e

$$\sum \frac{1}{n^2+1} \leq \sum \frac{1}{n^2} < +\infty.$$

So it converges absolutely

2 (a) Using ratio test, $\frac{n+1^{th} \text{ term}}{n^{th} \text{ term}} = \frac{\frac{x^{n+1}}{n+1}}{\frac{x^n}{n}} = \frac{x \cdot n}{n+1}$.

$$\lim \frac{a_{n+1}}{a_n} = x.$$

So for $|x| < 1$, it converges.
for $|x| > 1$, it diverges. $\}$ by ratio test

For $x = 1$, i.e. $\sum \frac{1}{2n}$ diverges

For $x = -1$. i.e $\sum \frac{1}{2n} (-1)^n$ converges by alternating sum test

$(-1, 1)$

(b) By root test $\lim_{n\to+\infty} \sqrt[n]{\frac{n^2}{2^n}} = \lim_{n\to+\infty} \frac{\sqrt[n]{n^2}}{2} = \frac{1}{2}$.

so for $|x| < 2$, it converges.
for $|x| > 2$, it diverges.

For $x = 2$, i.e $\sum \frac{(-1)^n n^2 2^n}{2^n} = \sum (-1)^n n^2$ diverges.

for $x = -2$, i.e $\sum n^2$ also diverges.

$\underline{(-\frac{1}{2}, \frac{1}{2})}$

(c) By root test, $\lim_{n\to+\infty} \sqrt[n]{\frac{2^n}{\sqrt{n}}} = \frac{2}{\sqrt[2n]{n}} = 2$

So for $|x+3| < \frac{1}{2}$ i.e $x \in (2\frac{1}{2}, 3\frac{1}{2})$, it converges.

For $|x+3| > \frac{1}{2}$, i.e $x \in (-\infty, 2\frac{1}{2}) \cup (3\frac{1}{2}, +\infty)$ it diverges.

For $x = 3\frac{1}{2}$, i.e. $\sum \frac{1}{\sqrt{n}}$ diverges.

for $x = 2\frac{1}{2}$, i.e. $\sum \frac{1}{\sqrt{n}} (-1)^n$ converges by alternating sum test.

$\underline{[2\frac{1}{2}, 3\frac{1}{2})}$

(d) By ratio test,

$$\frac{\frac{(n+1)!\,(x-7)^{n+1}}{2^{n+1}}}{\frac{n!\,(x-7)^n}{2^n}} = \frac{(n+1)(x-7)}{2} \longrightarrow +\infty$$

as $n \to +\infty$. $\forall x$.

i.e. the radius of convergence is 0.
So the only value for it to converge is $\underline{x=7}$.