$f(0) = -2$

1. (a) $f'(0) = 1$

$f''(0) = 0 \qquad f'''(0) = -2 \cdot 3! \qquad f''''(0) = 4!$

thus $\sum \frac{f^{(k)}(0)}{k!} x^k = -2 + x - 2x^3 + x^4$

$f(1) =$

(b) $f'(x) = 4x^3 - 6x^2 + 1 \qquad f'(1) = -1$

$f''(x) = 12x^2 - 12x \qquad f''(1) = 0$

$f'''(x) = 24x - 12. \qquad f'''(1) = 12$

$f''''(x) = 24 \qquad f''''(1) = 24$

$f(1) = -2$

So $-2 - 1(x-1) + \frac{12}{3!}(x-1)^3 + \frac{24}{4!}(x-1)^4$

Remark: for (a) (b) the answer are both $f(x)$ itself.

(c) $\cos x = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}(-1)^n \qquad x \in (-\infty, +\infty)$

so $\cos 3x^2 = \sum_{n=0}^{\infty} \frac{(3x^2)^{2n}}{(2n)!}(-1)^n$

(d) For $\ln(1+x)$ : $\sum_{n=1}^{\infty} \frac{x^n}{n}(-1)^{n+1}, \qquad x \in (-1, 1]$

thus for $\frac{1}{2}\ln(2x+1)$ : $\frac{1}{2}\sum_{n=1}^{\infty} \frac{(2x)^n}{n}(-1)^{n+1}.$

2 (a) $(-2 + x - 2x^3 + x^4)' = 1 - 6x^2 + 4x^3.$

(b) $-x + 6(x-1)^2 + 4(x-1)^3$

(c) $f(x) = \sum \frac{(-1)^n 9^n x^{4n}}{(2n)!}$

$f'(x) = \sum \frac{(-1)^n 9^n 4n \cdot x^{4n-1}}{(2n)!}$

(d)

$$f'(x) = \frac{1}{2}\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 2^n x^{n-1} \cdot \cancel{x}}{\cancel{x}} = \sum_{n=1}^{\infty} \frac{1}{2} \cdot (-1)^{n+1} 2^n x^{n-1}$$

3 (a) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

so $e^1 = \sum_{n=0}^{\infty} \frac{1}{n!}$

(b) $e^{-1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$

(c) $\sin x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} (-1)^n$

$$\sin \frac{\pi}{\sqrt{2}} = \sum_{n=0}^{\infty} \frac{\frac{\pi^{2n+1}}{2^{\frac{2n+1}{2}}}}{(2n+1)!} (-1)^n$$

$$= \sum_{n=0}^{\infty} \frac{\pi^{2n+1}}{2^{n+\frac{1}{2}} (2n+1)!} (-1)^n$$

so $\sqrt{2} \sin \frac{\pi}{\sqrt{2}} = \sum_{n=0}^{\infty} \frac{\pi^{2n+1}}{2^n (2n+1)!}$

(d) $\cos x = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} (-1)^n$

thus $\underset{\underset{-1}{\parallel}}{\cos 3\pi} = \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n} \pi^{2n}}{(2n)!}$

3. $(\arctan x)' = \frac{1}{1+x^2}$

$$= \sum_{n=0}^{\infty} x^{2n} (-1)^n \quad , \quad \forall x \in (-1, 1).$$

So the Taylor series for $\arctan x$ is

$$\int \frac{1}{1+x^2} dx = \int \sum_{n=0}^{\infty} x^{2n} (-1)^n dx$$

$$\xlongequal[\text{term by term}]{\text{integration}} \sum_{n=0}^{\infty} \int x^{2n} (-1)^n dx$$

$$= \sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1} (-1)^n$$

So $\arctan 1 = \sum_{n=0}^{\infty} \frac{1}{2n+1} (-1)^n.$

$\frac{\pi}{4}$ " $= 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \cdots$