MATH 104 – Practice Final Exam - Spring 2018

Name ____

1. The base of a solid S is the region in the first quadrant bounded by the curve $y = 1 - x^2$. The cross-sections perpendicular to the x-axis are squares. Compute the volume of the solid S.

(A) $\frac{1}{2}$ (D) $\frac{5}{9}$
(B) $\frac{3}{5}$ (E) $\frac{7}{12}$
(C) $\frac{4}{7}$ (F) $\frac{8}{15}$

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2. Consider the region D bounded by $y = x^2$ and $y = x + 2$. Compute the volume of solid obtained by revolving D about the x-axis.

(A) 5π (D) 24π
(B) $\frac{30}{7}\pi$ (E) 14π
(C) $\frac{72}{5}\pi$ (F) $\frac{36}{9}\pi$

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3. Find the length of the curve

$$y = \frac{e^{2x}}{2} + \frac{e^{-2x}}{8}, \qquad 0 \leqslant x \leqslant 2$$

(A) $\frac{1}{8}(e^4 + e^{-4})$ (D) $\frac{13}{12}$
(B) 1 (E) $4e^4 - \frac{1}{4}e^{-4}$
(C) $\frac{1}{8}(4e^4 - e^{-4} - 3)$ (F) $\frac{15}{8}$

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4. Solve the initial value problem

$$x\frac{dy}{dx} = y + x^3 e^x\,, \qquad y(1) = 0$$

and find $y(3)$.

(A) 1 (D) $3e^3$
(B) 3 (E) $6e^3$
(C) e^3 (F) $3e^3 + 2e^{-3}$

5. A tank initially contains 200 gallons of brine in which 50 lb of salt are dissolved. A brine containing 1 lb/gal of salt runs into the tank at the rate of 20 gallons per minute. During the process, the tank is kept well-mixed and the resulting salt water flows out of the tank at the rate of 20 gallons per minute. Find the amount of salt in the tank $\left(10\ln\frac{3}{2}\right)$ minutes after the process starts.

(A) 100
(B) 120
(C) 140
(D) $10\ln 10$
(E) $20\ln 3$
(F) $30\ln 2$

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6. $\displaystyle\int_0^{\pi/2} \cos^3 x\, \sin^2 x\, dx =$

(A) $\frac{2}{15}$
(B) $\frac{1}{5}$
(C) $\frac{4}{15}$
(D) $\frac{1}{3}$
(E) $\frac{2}{5}$
(F) $\frac{8}{15}$

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7. $\displaystyle\int_0^{\infty} xe^{-2x}\, dx =$

(A) 2
(B) 1
(C) $1/2$
(D) $1/4$
(E) 0
(F) The integral diverges.

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8. $\displaystyle\int_0^{1} \frac{2(x-1)}{(x+1)(x-3)}\, dx =$

(A) $4\ln 2 - 2\ln 3$
(B) $2\ln 3 - 2\ln 2$
(C) $2\ln 2 - \ln 3$
(D) $2\ln 3 + \ln 2$
(E) $3\ln 3 - \ln 2$
(F) $3\ln 2 - \ln 3$

9. $\int_0^4 \sqrt{4+x^2}\,dx =$

(A) $4\sqrt{5} - 2\ln(\sqrt{5}-2)$

(B) $4\sqrt{5} + 2\ln(\sqrt{5}-2)$

(C) $4\sqrt{5} - 2\ln(\sqrt{5}+2)$

(D) $4\sqrt{5} + 2\ln(\sqrt{5}+2)$

(E) $2\sqrt{5} + \ln(\sqrt{5}-2)$

(F) $2\sqrt{5} - \ln(\sqrt{5}+2)$

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10. The function

$$f(x) = \begin{cases} \ln x & 1 \leqslant x \leqslant e \\ 0 & \text{otherwise} \end{cases}$$

is a probability distribution (on the interval $[1, e]$). What is the mean of this distribution?

(A) $\frac{1}{2}(e^2+1)$

(B) $\frac{1}{4}(e^2+1)$

(C) $\frac{1}{2}(e^2-1)$

(D) $\frac{1}{4}(e^2-1)$

(E) $\frac{1}{2}(e+1)$

(F) $\frac{1}{4}(e+1)$

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11. Find the sum $A+B+C$, where

$$A = \lim_{n\to+\infty} \arctan\left(\ln\left(n^2+1\right)\right), \quad B = \lim_{n\to+\infty} e^{e^{-n^2}}, \quad C = \lim_{n\to+\infty} \frac{1}{1+\frac{1}{1+\frac{1}{n}}}$$

(A) $\frac{\pi}{2}$

(B) $\frac{3+\pi}{2}$

(C) $\frac{3}{2}$

(D) 0

(E) 1

(F) $\frac{1}{2}$

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12. Consider the following infinite series:

$$A = \sum_{n=1}^{\infty} \frac{n \ln n}{n^3+1}, \quad B = \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}, \quad C = \sum_{n=1}^{\infty} \frac{n^4}{2n!}$$

(A) A diverges, B and C converge

(B) B diverges, A and C converge

(C) C diverges, A and B converge

(D) A, B, and C diverge

(E) A, B, and C converge

(F) None of the above

13. It is estimated that humans released 40 billion tons of CO_2 into the environment last year. Suppose that hummankind will be capable of reducing by 10% its CO_2 emissions each year from now on. After a very long time, about how much CO_2 will humans have released to the environment since (and including) last year's 40 billion tons?

(A) 100 billion tons of CO_2

(B) 200 billion tons of CO_2

(C) 300 billion tons of CO_2

(D) 400 billion tons of CO_2

(E) 500 billion tons of CO_2

(F) None of the above

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14. Find the (largest) interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(x+3)^n}{2^n\, n^{1/2}}$$

(A) $[-5,-1]$

(B) $[-5,-1)$

(C) $(-5,-1)$

(D) $[1,5]$

(E) $[1,5)$

(F) $(1,5)$

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15. Let $F(x)$ be the unique function that satisfies $F(0) = 0$ and $F'(x) = x^3 \cos(2x)$ for all x. Find the Taylor Series of $F(x)$ centered at $x_0 = 0$.

(A) $\sum_{n=0}^{\infty} \frac{(-1)^n 4^n x^{2n+4}}{(2n+4)(2n)!}$

(B) $\sum_{n=0}^{\infty} \frac{(-1)^n 2^n x^{2n+2}}{(2n+4)(2n)!}$

(C) $\sum_{n=0}^{\infty} \frac{(-1)^n 4^n x^{2n+4}}{(2n)!}$

(D) $\sum_{n=0}^{\infty} \frac{(-1)^n 4^n x^{2n+3}}{(2n)!}$

(E) $\sum_{n=0}^{\infty} \frac{(-1)^n 2^n x^{2n}}{(2n+1)!}$

(F) $\sum_{n=0}^{\infty} \frac{(-1)^n 4^n (2n+4) x^{2n+3}}{(2n)!}$

I think the answers are FCCEA/ADCAB/BEDBA