# Homework Set 10

Due: Apr 13, 2017 (in class)

1. Compute the work done by the following force fields along the path $\gamma(t) = (\cos t, \sin t)$, where $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$:

   a) $\vec{F}(x, y) = (x, y)$
   b) $\vec{F}(x, y) = (x + y, 2xy - 3)$
   c) $\vec{F}(x, y) = (6xy - y^3, 3y^2 + 3x^2 - 3xy^2)$

2. A $50m$ long metal chain is hanging from the top of a building. Assuming that the acceleration due to gravity is $g = 10m/s^2$ and the density of the metal chain is constant at $2kg/m$, find the work required to lift $20m$ of this chain up to the top. (Note that $30m$ will remain hanging).

3. Let $\vec{F} = \nabla\varphi$, where $\varphi(x, y, z) = x^2 + y^2 + e^z$. Compute the following line integrals:

   a) $I_1 = \displaystyle\int_\gamma \vec{F}\,\mathrm{d}\gamma$ where $\gamma$ is any path from $(0, 0, 0)$ to $(1, 1, 1)$
   b) $I_2 = \displaystyle\int_\gamma \vec{F}\,\mathrm{d}\gamma$ where $\gamma$ is any path from $(1, 1, 1)$ to $(2, 0, 1)$
   c) $I_3 = \displaystyle\int_\gamma \vec{F}\,\mathrm{d}\gamma$ where $\gamma$ is any path from $(2, 0, 1)$ to $(0, 0, 0)$

   Compute the sum $S = I_1 + I_2 + I_3$ and explain why the value of $S$ could have been determined without any computations.

4. For the following vector fields $\vec{F}$, decide whether there exists $\varphi\colon \mathbb{R}^3 \to \mathbb{R}$ such $\vec{F} = \nabla\varphi$. If yes, then find $\varphi$.

   a) $\vec{F} = (x, y, z)$
   b) $\vec{F} = (e^{xz}, x + y + z, 1)$
   c) $\vec{F} = (2xy, x^2 + \cos(y), 0)$
   d) $\vec{F} = (z^2, x + y, 4\sin(xz))$

5. Compute the following line integrals using Green's theorem ($\gamma$ is assumed to be oriented counterclockwise):

   a) $\displaystyle\int_\gamma x^2 y\,\mathrm{d}x + xy^3\,\mathrm{d}y$ where $\gamma$ is the square with vertices $(0, 0)$, $(0, 1)$, $(1, 0)$, $(1, 1)$
   b) $\displaystyle\int_\gamma (x+2y)\,\mathrm{d}x + (x-2y)\,\mathrm{d}y$ where $\gamma$ is the curve determined by the arc of parabola $y = x^2$ from $(0, 0)$ to $(1, 1)$ and the line segment joining the same two points
   c) $\displaystyle\int_\gamma x^2\,\mathrm{d}x + y^2\,\mathrm{d}y$ where $\gamma$ is the curve determined by $x^6 + y^6 = 1$