Homework Set 8

DUE: MAR 30, 2017 (IN CLASS)

1. (From HW 7) Use polar coordinates to compute the following double integrals:

   (a) $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{(1+x^2+y^2)^2}\,\mathrm{d}y\,\mathrm{d}x$

   (b) $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \ln(1+x^2+y^2)\,\mathrm{d}x\,\mathrm{d}y$

2. What is the area of the region in the plane bounded by the curve given in polar coordinates by $r(\theta) = 4 + 2\cos(2\theta)$?

3. Compute the following triple integrals:

   a) $\displaystyle\int_{0}^{2}\int_{0}^{1}\int_{x^2}^{1} 12xze^{zy^2}\,\mathrm{d}y\,\mathrm{d}x\,\mathrm{d}z$

   b) $\displaystyle\int_{0}^{2}\int_{0}^{4-x^2}\int_{0}^{x} \frac{\sin(2z)}{4-z}\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}x$

   c) $\displaystyle\int_{-1}^{1}\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2z}{(1+x^2+y^2)^2}\,\mathrm{d}y\,\mathrm{d}x\,\mathrm{d}z$

4. Compute the total mass and the center of mass of a plate in the shape of a disk $D = \{x^2 + y^2 \leq 4\}$ whose density at the point (x, y) is given by $\delta(x, y) = x + 3$.

5. Find the volume of the region given by the intersection of the solid cylinders $x^2+y^2 \leq 1$ and $x^2 + z^2 \leq 1$.
   HINT: Compute first the volume of part of the region that lies in the positive octant, which is one eighth of the total volume.

6. BONUS PROBLEM (OPTIONAL) What is the probability that the quadratic polynomial $f(x) = ax^2 + bx + c$ has real roots if a, b, c are randomly chosen (with uniform distribution) in the interval $[0, 1]$?
   HINT: $f(x)$ has real roots if and only if $\Delta = b^2 - 4ac \geq 0$.