Math 114, Spring 2017

Practice Problems for Midterm 1

1. Write the equation of the sphere centered at $P_1 = (1, 0, 1)$ with radius $r_1 = 2$ and of the sphere centered at $P_2 = (0, 2, 0)$ with radius $r_2 = 1$. Determine the relative position of these spheres, that is, whether one is contained inside the other, or they intersect (at one or more points), or they are disjoint.

2. Let π_1 be the plane through the origin which is spanned by the vectors $\vec{v} = (1, 1, 0)$ and $\vec{w} = (1, -1, 0)$. Let π_2 be the plane that contains the points $P = (0, 1, 4)$, $Q = (-2, 3, 4)$ and $R = (1, \sqrt{2}, 4)$. What is the distance between π_1 and π_2?

   Note: The *distance* between two planes is the shortest possible distance between points in each of the planes

3. Consider the following curves in space:

   $\alpha(t) = (-1+\cos t, \sin t, -t) \qquad \beta(t) = (t, t^2, t^3) \qquad \gamma(t) = (t^4 - t^2, -1 + e^t \cos t, 1 - e^{2t})$

   Compute the volume of the parallelepiped spanned by the tangent vectors $\alpha'(0)$, $\beta'(0)$, and $\gamma'(0)$ of these curves at $t = 0$.

4. Consider the quadric surface given by $z = 2x^2 - 4y^2$. Classify the conics obtained by intersecting this surface with each of the coordinate planes (xy-plane, xz-plane, and yz-plane). Sketch these planar sections (but you do not need to sketch the surface).

5. Compute the arc length of the curve parametrized by

   $$x(t) = t^{3/2} \quad y(t) = (1-t)^{3/2},$$

   for $0 \le t \le 1$, and compute the curvature of the curve at $t = \frac{1}{2}$.

6. Imagine a strange world where Newton's second law doesn't hold, but you know that the *acceleration* of a projectile with trajectory $\vec{\gamma}(t)$ is given by $\vec{a}(t) = -6t\,\vec{j}\ m/s^2$, where $\vec{j}$ stands for the vertical direction. This projectile is fired from ground level with an initial velocity of 10 m/s in the horizontal direction and 49 m/s in the vertical direction. How far has the projectile travelled horizontally when it strikes the ground?

7. Consider the curve parametrized by

   $$x(t) = \sqrt{2}\sin t \quad y(t) = 1 - \cos t \quad z(t) = 1 + \cos t.$$

   Find the unit vectors $\vec{T}$, $\vec{N}$, $\vec{B}$ (orthonormal moving frame) at $t = 0$.

8. Find the parametrization $\vec{r}(t)$ of a curve given that

$$\frac{\mathrm{d}^2\vec{r}}{\mathrm{d}t^2}(t) = (e^t, \sin t, t)$$

and

$$\frac{\mathrm{d}\vec{r}}{\mathrm{d}t}(0) = (0, 0, 1)$$

and

$$\vec{r}(1) = (e, 1, 1).$$

9. Consider the curve given in polar coordinates by $r = -4\sin\theta$. Write an expression for this curve in Cartesian coordinates (that is, x and y) and recognize what conic it is.

   Hint: Multiply both sides of the equation in polar coordinates by r.