Vinson Liao

Midterm 2 Review

1. Find the value of the limit:

$$\lim_{(x,y)\to(1,1)} \frac{x^3y^3 - 1}{xy - 1}$$

2. The tangent plane to surface at (0,1,2) intersects the y-axis at what point?

$$\cos(\pi x) - x^2y + e^{xz} + yz = 4$$

3. Let $f(x,y) = \frac{x-y}{x+y}$
   Find the directions u such that $D_u f\left(\frac{-1}{2}, \frac{3}{2}\right) = 1$

4. Integrate

$$\int_0^1 \int_x^{\sqrt{2-x^2}} (x+y)\sqrt{x^2+y^2}\, dy\, dx$$

5. Find and classify the critical points

$$f(x,y) = x - y^2 - \ln(x+y)$$

6. A mountain's height is given as $h(x,y) = e^{x/y}$. What direction should you walk (infinitesimally small) to keep height constant?

7. Find the minimum distance from the ellipsoid $4 = x^2 + 4y^2 + 2z^2$ to the plane $x + 2y + z = 6$.
   Hint: This is a Lagrange question. You will need to determine what your function to minimize is (this may require stuff from the beginning of the semester) and what the constraint for this function is.

8. Sketch the domain of

$$f(x,y) = \frac{\ln(y-x)}{\sqrt{x-y+4}}$$

9. Integrate

$$\iint_R \arctan\left(\frac{y}{x}\right) e^{-\sqrt{x^2+y^2}} dA$$

   Where R is the region in the first quadrant bounded by $1 \le x^2 + y^2 \le e$

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Potentially Useful Information to Remember:

$$\int xsinx\,dx = sinx - xcosx + C$$

$$\int xcosx\,dx = cosx + xsinx + C$$

$$\int xe^x\,dx = xe^x - e^x + C$$

$$\int xe^{-x}\,dx = -xe^{-x} - e^{-x} + C$$

$$sin^2 x = \frac{1 - \cos(2x)}{2}$$

$$cos^2 x = \frac{1 + \cos(2x)}{2}$$