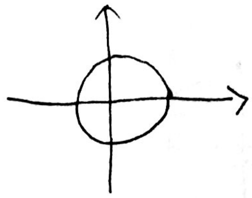
HW8 Solutions

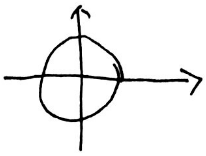
1. $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{(1+x^2+y^2)^2}\,dy\,dx \Rightarrow$



$\displaystyle\Rightarrow \int_0^{2\pi}\int_0^{1} \frac{2r}{(1+r^2)^2}\,dr\,d\theta =$

$\displaystyle= 2\pi\int_0^1 \frac{2r}{(1+r^2)^2}\,dr = \pi$

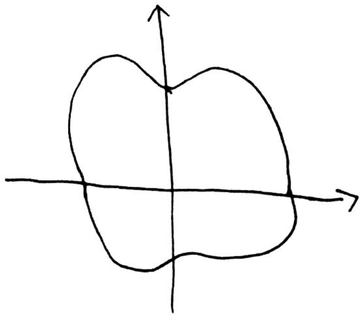
b. $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \ln(1+x^2+y^2)\,dx\,dy \Rightarrow$



$\displaystyle\Rightarrow \int_0^{2\pi}\int_0^{1} \ln(1+r^2)\,r\,dr\,d\theta =$

$\displaystyle= 2\pi\int_0^1 \ln(1+r^2)\,r\,dr\,d\theta = \Big|_0^1 = \pi\left[(r^2+1)\ln(r^2+1) - r^2\right]\Big|_0^1 = \pi\left[2\ln 2 - 1\right]$

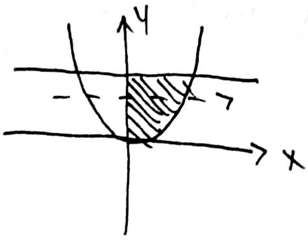
2. $r(\theta) = 4+2\cos(2\theta)$



$\displaystyle A = \iint dA = \int_0^{2\pi}\int_0^{4+2\cos(2\theta)} r\,dr\,d\theta = \frac{1}{2}\int_0^{2\pi}\left[4+2\cos(2\theta)\right]^2 d\theta =$

$\displaystyle= \frac{1}{2}\int_0^{2\pi}\Big(\underbrace{16}_{16\pi} + \underbrace{16\cos(2\theta)}_{0} + \underbrace{4\cos^2(2\theta)}_{2\pi}\Big)\,d\theta = 18\pi$

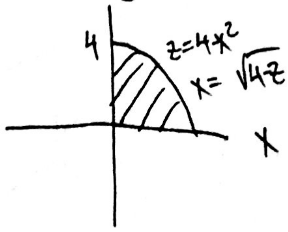
3. $\displaystyle\int_0^2\int_0^1\int_{x^2}^{1} 12xz\,e^{zy^2}\,dy\,dx\,dz \Rightarrow$



$\displaystyle= \int_0^2\int_0^1\int_0^{\sqrt{y}} 12xz\,e^{zy^2}\,dx\,dy\,dz = 6\int_0^2\int_0^1 yz\,e^{zy^2}\,dy\,dz = 6\int_0^2 \frac{1}{2}e^{zy^2}\Big|_0^1 dz = 3\int_0^2 (e^z - 1)\,dz =$

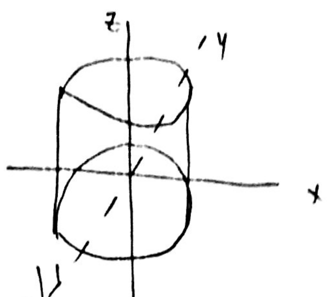
$\displaystyle= 3(e^z - z)\Big|_0^2 = 3e^2 - 2*3 - 3 = 3e^2 - 9$

6. $\displaystyle\int_0^2\int_0^{4-x^2}\int_0^{x} \frac{\sin(2z)}{4-z}\,dy\,dz\,dx = \int_0^2\int_0^{4-x^2} \frac{x\sin(2z)}{4-z}\,dz\,dx \Rightarrow$



$\displaystyle= \int_0^4\int_0^{\sqrt{4-z}} \frac{x\sin(2z)}{4-z}\,dz\,dx = \frac{1}{2}\int_0^4 \sin(2z)\,dz = -\frac{1}{4}\cos(2z)\Big|_0^4 = -\frac{1}{4}(\cos 8 - 1)$

c. $\displaystyle\int_{-1}^{1}\int_{-1}^{1}\int_{-\sqrt{1+x^2}}^{\sqrt{1+x^2}} \frac{2z}{(1+x^2+y^2)^2}\, dy\,dx\,dz \Rightarrow$



$=$

$$= \int_0^{2\pi}\int_0^{1}\int_{-1}^{1} \frac{2z}{(1+r^2)^2}\, r\,dr\,d\theta = \int_0^{2\pi}\int_0^{1} \frac{r}{(1+r)^2}\, z^2 \Big|_{-1}^{1} = 0$$

4. $\delta = x+3$; $D = \{x^2+y^2 \le 4\}$

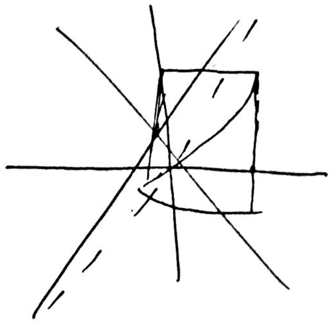
$$\text{mass} = \iint_D \delta\, dA = \int_0^{2\pi}\int_0^{2} (r\cos\theta + 3)\, r\,dr\,d\theta = \int_0^{2\pi} \frac{3}{2}r^2 + r^2\cos\theta \Big|_0^2 d\theta = \int_0^{2\pi} (6 + 4\cos\theta)\, d\theta = 12\pi$$

$$\bar{x} = \frac{1}{2\pi}\iint_D x\delta\, dA = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{2} (r^2\cos^2\theta + 3r\cos\theta)\, r\,dr\,d\theta = 2$$

$$\bar{y} = \frac{1}{2\pi}\iint_D y\delta\, dA = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{2} (r^2\cos\theta\sin\theta + 3r\sin\theta)\, r\,dr\,d\theta = 0$$

CoM $(2,0)$

5.



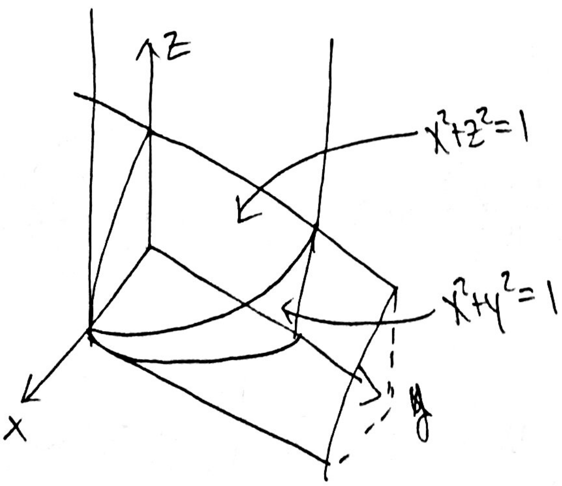
Sorry; suck @ drawing

Refer to PG 914 for picture

$$\frac{V}{8} = \iiint dV = \int_0^{}\int_0^{}\int_0^{\sqrt{1-x^2}} dz\,d\_\,d\_ =$$

$$= \int_0^{1}\int_0^{\sqrt{1-x^2}} \sqrt{1-x^2}\, dy\,dx = \int_0^{1} (1-x^2)\,dx = x - \frac{x^3}{3}\Big|_0^1 = \frac{2}{3}$$

$V = \frac{16}{3}$



6. $\Delta = b^2 - 4ac \geq 0$ $\qquad$ $f(x) = ax^2 + bx + c$

$b^2 \geq 4ac.$ $\qquad$ $a, b, c \in [0,1]$ uniform

* The box of $1 \times 1 \times 1$ is the total $a, b, c \in [0,1]$

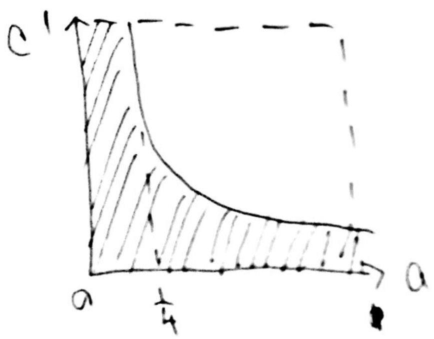
* The surface of $b^2 = 4ac$ + above is the boundary of when $b^2 \geq 4ac$.

* Volume inside box and above surface is the probability

Projection @ $b = 1$

$1 = 4ac$

$\frac{1}{4} = ac$



+ Integral must be broken up

$$\int_0^{\frac{1}{4}} \int_0^1 \int_{\sqrt{4ac}}^1 db\,dc\,da + \int_{\frac{1}{4}}^1 \int_0^{\frac{1}{4a}} \int_{\sqrt{4ac}}^1 db\,dc\,da =$$

$$= \frac{5}{36} + \frac{\ln(2)}{6}$$