Homework Set 5

Due: Mar 2, 2016 (in class)

1. Bretscher Section 5.1: Exercises 32, 33

2. Bretscher Section 5.2: Exercises 32, 33, 36

3. Bretscher Section 5.3: Exercises 45, 46, 60, 61

4. Bretscher Section 5.5: Exercises 10, 17, 18, 30

5. Let $V = \{ A \in M_{2 \times 2}(\mathbb{R}) : \langle A, \text{Id} \rangle_{HS} = 0 \}$ be the subspace of $2 \times 2$ real matrices that are orthogonal to the identity matrix with respect to the Hilbert-Schmidt inner product $\langle A, B \rangle_{HS} = \text{tr}(AB^t)$. Find an orthonormal basis of $(V, \langle \cdot, \cdot \rangle_{HS})$.

6. Given $A \in M_{n \times n}(\mathbb{R})$, recall that the operator norm of $A$ is given by

$$\| A \| = \max_{\| v \|=1} \| Av \|,$$

and its Hilbert-Schmidt norm is $\| A \|_{HS} = \sqrt{\langle A, A \rangle_{HS}}$. Prove that $\| A \| \leq \| A \|_{HS}$ for all diagonal matrices $A$. When does equality hold?

7. Challenge (2 bonus pts) Prove that $\| A \| \leq \| A \|_{HS}$ for any matrix $A \in M_{n \times n}(\mathbb{R})$, and also characterize when equality holds.