3.10 Related Rates

Goal:
Compute the rate of change of one quantity in terms of the rate of change of another quantity (which may be more easily measured).

1. Read the problem carefully.
2. Draw a diagram if possible.
3. Introduce notation. Assign symbols to all quantities that are functions of time.
4. Express the given information and the required rate in terms of derivatives.
5. Write an equation that relates the various quantities of the problem. If necessary, use the geometry of the situation to eliminate one of the variables by substitution (as in Example 3).
6. Use the Chain Rule to differentiate both sides of the equation with respect to \( t \).
7. Substitute the given information into the resulting equation and solve for the unknown rate.
A cylindrical tank with radius 5 m is being filled with water at a rate of 3 m³/min. How fast is the height of the water increasing?
A plane flying horizontally at an altitude of 1 mi. and a speed of 500 mi./hr. passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 mi. away from the station. 
ex. 3

At noon, ship A is 150 km. west of ship B. Ship A is sailing east at 35 km./hr. and ship B is sailing north at 25 km./hr. How fast is the distance between the ships changing at 4:00 pm?
A street light is mounted at the top of a 15 ft. tall pole. A man 6 ft. tall walks away from the pole with a speed of 5 ft./s. along a straight path. How fast is the tip of his shadow moving when he is 40 ft. from the pole?
Two sides of a triangle have lengths 12 m. and 15 m. The angle between them is increasing at a rate of 2° / min. How fast is the length of the third side increasing when the angle between sides of fixed length is 60°?
Water is leaking out of an inverted conical tank at a rate of 10,000 cm$^3$/min. at the same time that water is being poured into the tank at a constant rate. The tank has height 6 m. and the diameter at the top is 4 m. If the water level is rising at a rate of 20 cm./min. when the height of the water is 2 m., find the rate at which the water is being pumped into the tank.