Goal: To find a general formula for the equation of the tangent line.

The linearization of $f$ at $a$

Why?: To use the tangent line to approximate the values of the function.

The tangent line is a good approximation to the values of the function as long as you stay "close" to $x = a$. 

example:

\[ f(x) = \sqrt{x + 2} \]

Find the linear approximation of the function at \( a = 7 \).

\[
\frac{dy}{dx} = f'(x) \Rightarrow dy = f'(x) \, dx
\]

this is called the differential \( dy \)

\[
P(x, f(x)) \quad Q(x + \Delta x, f(x + \Delta x))
\]

\[
dy = f'(x) \, dx \quad \Delta y = f(x + \Delta x) - f(x)
\]

The smaller \( \Delta x \) gets, the closer \( dy \) is to \( \Delta y \).
Compare the values of $dy$ and $\Delta y$ for $f(x) = x^3 + x^2 - 2x + 1$ as $x$ changes from $x = 2$ to $x = 2.05$.

The radius of a sphere was measured and found to be 21 cm with a possible error in measurement of at most 0.05 cm.

What is the maximum error in using this value of the radius to compute the volume of the sphere?

Relative Error gives a better picture