4.3 How the derivative relates to the function

<table>
<thead>
<tr>
<th>$f(x)$</th>
<th>$f'(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>graph has positive sloping tangent lines</td>
<td>$f$ is increasing</td>
</tr>
<tr>
<td>graph has negative sloping tangent lines</td>
<td>$f$ is decreasing</td>
</tr>
<tr>
<td>graph has a tangent line with a slope of 0 (horizontal tangent line)</td>
<td>$f$ is neither increasing or decreasing</td>
</tr>
</tbody>
</table>

The First Derivative Test

Purpose: to find and classify local maximum and local minimum values

Let $f(x)$ be a continuous function at $x = c$ and let $c$ be a critical number of $f$.

- $f'(c) > 0$  \Rightarrow  $f$ has a local maximum at $c$
- $f'(c) < 0$  \Rightarrow  $f$ has a local minimum at $c$
Example: Use the first derivative test to find the local maximum and local minimum values.

\[ f(x) = x^3 - 5x + 3 \]

1) Find the critical numbers of \( f \).

\[ f'(x) \]

↓ ↓

\( f \) has a local \( f \) has a local

at \( x = \) at \( x = \)
4.3 How the second derivative relates to the function

**Concavity** – describes the ______________________

When the curve lies above the tangents of \( f \) on an interval \((a,b)\), we say that \( f \) is **concave** _____ or concave ___ for short.

When the curve lies below the tangents of \( f \) on an interval \((a,b)\), we say that \( f \) is **concave** _______ or concave ___ for short.

\[ f''(x) \text{ is } _____ \quad f''(x) \text{ is } _____ \]

When \( f \) changes concavity at a point, we call that point an ______________.
The Second Derivative Test

Purpose: to find and classify local maximum and local minimum values

Let \( f'' \) be continuous near \( x = c \).

\[
\begin{align*}
\Rightarrow f & \text{ has a local maximum at } c \\
\Rightarrow f & \text{ has a local minimum at } c
\end{align*}
\]

Use the second derivative test when it isn't that much trouble to take the second derivative.

Example: Use the second derivative test to find the local maximum and local minimum values.

\[ f(x) = x^3 - 5x + 3 \]

\[
\begin{align*}
\downarrow \quad \downarrow \\
\text{f has a local} & \text{ at } x = \quad \text{f has a local} \\
\text{maximum} & \text{ at } x = \quad \text{minimum} & \text{ at } x =
\end{align*}
\]
\[ f(x) = (x^2 - 1)^{\frac{3}{2}} \]

Find the intervals where \( f(x) \) is increasing and decreasing.
Find all local maximum and local minimum values.
Find the intervals where \( f(x) \) is concave up and concave down.
Find all inflection points.

\[ f'(x) \text{ is a polynomial, it exists for all } x \Rightarrow \text{it is never undefined.} \]

These are the critical numbers of \( f \)

\[
\begin{array}{c|c}
 f'(x) & \text{description} \\
\hline
- & f \text{ is decreasing on} \\
\downarrow & f \text{ is increasing on} \\
\hline
\end{array}
\]

\( f \) has a \text{local} \ \underline{\text{value of \ 0 \ at \ \ x = -1}} \]

\[ f(x) = (x^2 - 1)^{\frac{3}{2}} \]

\[ f' = \text{inflection points} \]

\( f \) is \text{concave up on} \\
\( f \) is \text{concave down on}