4.6 Optimization

**Steps in Solving Optimization Problems**

1. **Understand the Problem**  The first step is to read the problem carefully until it is clearly understood. Ask yourself: What is the unknown? What are the given quantities? What are the given conditions?

2. **Draw a Diagram**  In most problems it is useful to draw a diagram and identify the given and required quantities on the diagram.

3. **Introduce Notation**  Assign a symbol to the quantity that is to be maximized or minimized (let’s call it $Q$ for now). Also select symbols ($a$, $b$, $c$, $x$, $y$) for other unknown quantities and label the diagram with these symbols. It may help to use initials as suggestive symbols—for example, $A$ for area, $h$ for height, $t$ for time.

4. **Express $Q$ in terms of some of the other symbols from Step 3.**

5. **If $Q$ has been expressed as a function of more than one variable in Step 4, use the given information to find relationships (in the form of equations) among these variables. Then use these equations to eliminate all but one of the variables in the expression for $Q$. Thus $Q$ will be expressed as a function of one variable $x$, say, $Q = f(x)$. Write the domain of this function.**

6. **Use the methods of Sections 4.1 and 4.3 to find the absolute maximum or minimum value of $f$. In particular, if the domain of $f$ is a closed interval, then the Closed Interval Method in Section 4.1 can be used.**
12. A farmer wants to construct a pen next to a barn 60 feet long, using all of the barn as part of one side of the pen. Find the dimensions of the pen with the largest area that the farmer can build if 300 feet of fencing material is available. There is no fence along the barn wall.

A poster has printed material in the middle and a border around it. The poster must contain 60 sq. cm. of printed material. The left and right margins should be 5 cm., the top and bottom margins should be 3 cm. What should be the dimensions of the printed material in order to minimize the amount of paper used for the poster?
A box with a square base and an open top must have a volume of 4000 cubic cm. Find the dimensions of the box that minimize the amount of material being used.

Find the points on the ellipse \(4x^2 + y^2 = 4\) that are furthest away from the point (1,0).

General ellipse equation:
\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
\]
A Norman window has the shape of a rectangle surmounted by a semi-circle. If the perimeter of the window is 30 ft., find the value of \( x \) so that the greatest possible amount of light is admitted.