Hyperbolic sine:
\[ \sinh x = \frac{e^x - e^{-x}}{2} \]

Hyperbolic cosine:
\[ \cosh x = \frac{e^x + e^{-x}}{2} \]
7.3 Hyperbolic Trig. Functions

Hyperbolic tangent:
\[ \tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \]

Hyperbolic cotangent:
\[ \coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}} \]

Hyperbolic secant:
\[ \text{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}} \]

Hyperbolic cosecant:
\[ \text{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}} \]
### TABLE 7.5 Derivatives of hyperbolic functions

\[
\begin{align*}
\frac{d}{dx} (\sinh u) &= \cosh u \frac{du}{dx} \\
\frac{d}{dx} (\cosh u) &= \sinh u \frac{du}{dx} \\
\frac{d}{dx} (\tanh u) &= \text{sech}^2 u \frac{du}{dx} \\
\frac{d}{dx} (\coth u) &= -\text{csch}^2 u \frac{du}{dx} \\
\frac{d}{dx} (\text{sech} u) &= -\text{sech} u \tanh u \frac{du}{dx} \\
\frac{d}{dx} (\text{csch} u) &= -\text{csch} u \coth u \frac{du}{dx}
\end{align*}
\]

### TABLE 7.6 Integral formulas for hyperbolic functions

\[
\begin{align*}
\int \sinh u \, du &= \cosh u + C \\
\int \cosh u \, du &= \sinh u + C \\
\int \text{sech}^2 u \, du &= \tanh u + C \\
\int \text{csch}^2 u \, du &= -\coth u + C \\
\int \text{sech} u \tanh u \, du &= -\text{sech} u + C \\
\int \text{csch} u \coth u \, du &= -\text{csch} u + C
\end{align*}
\]

### TABLE 7.4 Identities for hyperbolic functions

\[
\begin{align*}
\cosh^2 x - \sinh^2 x &= 1 \\
\sinh 2x &= 2 \sinh x \cosh x \\
\cosh 2x &= \cosh^2 x + \sinh^2 x \\
\cosh^2 x &= \frac{\cosh 2x + 1}{2} \\
\sinh^2 x &= \frac{\cosh 2x - 1}{2} \\
\tanh^2 x &= 1 - \text{sech}^2 x \\
\coth^2 x &= 1 + \text{csch}^2 x
\end{align*}
\]
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7.3 Hyperbolic Trig. Functions

Why “hyperbolic”?

Why “hyperbolic”? Imagine a cable, like a telephone line or TV cable, strung from one support to another and hanging freely. The cable’s weight per unit length is a constant \( w \) and the horizontal tension at its lowest point is a vector of length \( H \). If we choose a coordinate system for the plane of the cable in which the \( x \)-axis is horizontal, the force of gravity is straight down, the positive \( y \)-axis points straight up, and the lowest point of the cable lies at the point \( y = \frac{H}{w} \) on the \( y \)-axis (see accompanying figure), then it can be shown that the cable lies along the graph of the hyperbolic cosine

\[
y = \frac{H}{w} \cosh \frac{w}{H} x.
\]

Such a curve is sometimes called a **chain curve** or a **catenary**, the latter deriving from the Latin *catena*, meaning “chain.”

Where is this used?

**Hanging cables** Imagine a cable, like a telephone line or TV cable, strung from one support to another and hanging freely. The cable’s weight per unit length is a constant \( w \) and the horizontal tension at its lowest point is a vector of length \( H \). If we choose a coordinate system for the plane of the cable in which the \( x \)-axis is horizontal, the force of gravity is straight down, the positive \( y \)-axis points straight up, and the lowest point of the cable lies at the point \( y = \frac{H}{w} \) on the \( y \)-axis (see accompanying figure), then it can be shown that the cable lies along the graph of the hyperbolic cosine

\[
y = \frac{H}{w} \cosh \frac{w}{H} x.
\]

Such a curve is sometimes called a **chain curve** or a **catenary**, the latter deriving from the Latin *catena*, meaning “chain.”

Laplace’s Equation

\[
u_{xx} + u_{yy} = 0
\]

Used in many areas of physics:

- electromagnetic theory
- heat transfer
- fluid dynamics
- special relativity