A power series is a series of the form
\[ \sum_{n=0}^{\infty} c_n x^n = \]
where:
a) 
b)

For each fixed \( x \), the series above is a series of constants that we can test for convergence or divergence.

A power series may converge for some values of \( x \) and diverge for other values of \( x \).

The sum of the series is a function

whose ________ is the set of all \( x \) for which the series converges. 
\( f(x) \) is reminiscent of a ________ but it has infinitely many terms.

If all \( c_n \)'s = 1, we have
\[ f(x) = 1 + x + x^2 + \ldots + x^n + \ldots = \sum_{n=0}^{\infty} x^n \]
This is the ________________ with ____.
The power series will converge for ____ and diverge for all other \( x \).
In general, a series of the form

is called a power series ________ or a power series about \( a \)

We use the \( \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = L \) to find for what values of \( x \) the series converges.

Solve for \( |x - a| \) to get \( |x - a| < R \)

\[ \Rightarrow -R < x - a < R \]
\[ \Rightarrow a - R < x < a + R \]

This is called the ________ ________ (I.O.C.).

Plug in the endpoints to check for convergence or divergence at the endpoints.

Find the radius of convergence and the interval of convergence.

\[ \sum_{n=1}^{\infty} \frac{(-1)^n n^2 x^n}{2^n} \]

\( x = \) ________ \( x = \) ________

Radius of convergence: ________
Interval of convergence: ________
Find the radius of convergence and the interval of convergence.

\[
\sum_{n=1}^{\infty} \frac{3^n (x + 4)^n}{\sqrt{n}}
\]

Check endpoints:

\[x = \quad x = \]

R.O.C.: ______
I.O.C.: ______

Find the radius of convergence and the interval of convergence.

\[
\sum_{n=1}^{\infty} \frac{(4x + 1)^n}{n^2}
\]

Check endpoints:

\[x = \quad x = \]

R.O.C.: ______
I.O.C.: ______
Sometimes the Root Test can be used just as the Ratio Test.

When \( a_n \) can be written as \( (b_n)^n \), then the Root Test should be used.

\[
\sum_{n=1}^{\infty} \frac{3^n (x - 5)^n}{n^n}
\]

\[
\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = 0 \Rightarrow
\]

\[
\sum_{n=1}^{\infty} \frac{n! (x - 7)^n}{2^n}
\]

\[
\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \infty \Rightarrow
\]
Find the radius of convergence.

\[
\sum_{n=1}^{\infty} \frac{(-1)^n (n!)^2 x^{2n}}{(2n)!}
\]

Radius of convergence: _____