Average of $n$ numbers:
$$\frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$$

Take the $n$ numbers to be sample points for a function:
$$\frac{f(x_1^*) + f(x_2^*) + f(x_3^*) + \cdots + f(x_n^*)}{n}$$

Partition the interval $[a, b]$ into $n$ subintervals of equal length.

Here $n = 8$.

What is the width of each subinterval? Call this $\Delta x$.

We now have:
$$f(x_1^*) + f(x_2^*) + f(x_3^*) + \cdots + f(x_n^*)$$

Taking the limit as $n \to \infty$, we get
$$\lim_{n \to \infty} \frac{\sum_{i=1}^{n} f(x_i^*) \Delta x}{b-a}$$

**Average value of** $f(x)$ on an interval $[a, b]$

Find the average value of the function $\frac{3}{(1+r)^2}$ on the interval $[1, 6]$. 
Mean Value Theorem for Integrals

Let $f$ be ________ on $[a,b]$, then there is a value $c$ in $[a,b]$ such that

$$f(c) = \text{or} \quad f(c)(b-a) = \int_{a}^{b} f(x) \, dx = \text{ }$$

Proof:
Let $f$ be continuous on $[a,b]$, By the ________ Value Theorem, there is a $m$ and $M$ such that

Then $\quad \leq \int_{a}^{b} f(x) \, dx \leq \$ $\Rightarrow$

By the ________ Value Theorem, there is a $c$ in $[a,b]$ with $f(c) = \frac{1}{b-a} \int_{a}^{b} f(x) \, dx$. 
(a) Find the average value of $f(x) = \sqrt{x}$ on the interval $[0, 4]$.

(b) Find $c$ such that $f_{ave} = f(c)$.

(c) Sketch the graph of $f$ and a rectangle whose area is the same as the area under the graph of $f$. 