**Area of the band**

\[
Area = 2\pi \left( \frac{y_{i+1} + y_i}{2} \right) d\left(P_{i+1}P_i\right)
\]

\[
= 2\pi f'(x^*) \sqrt{1 + \left(f'(x^*)\right)^2} \Delta x
\]

Total surface area = \[\sum_{i=0}^{n-1} 2\pi f'(x^*) \sqrt{1 + \left(f'(x^*)\right)^2} \Delta x\]

Total surface area = \[\lim_{n \to \infty} \sum_{i=0}^{n-1} 2\pi f'(x^*) \sqrt{1 + \left(f'(x^*)\right)^2} \Delta x\]

Surface Area = \[\int_a^b 2\pi f(x) \sqrt{1 + \left(f'(x)\right)^2} \, dx\]

For the radius, take an average.
For the length, take the distance from \(P_{i+1}\) to \(P_i\).

In 9.1 we saw \(d(P_i, P) = \sqrt{1 + \left(f'(x^*)\right)^2} \Delta x\)
For small \(\Delta x\), \(y_i = f(x_i) = f(x^*)\)
and \(y_{i-1} = f(x_{i-1}) = f(x^*)\)

---

A function with a continuous derivative on \([a, b]\) \(\Rightarrow\) the area of surface obtained by rotating the graph of a function about the \(y\)-axis for \(a \leq x \leq b\) is

\[
SA = 2\pi \int_a^b x \, ds
\]

A function with a continuous derivative on \([a, b]\) \(\Rightarrow\) the area of surface obtained by rotating the graph of a function about the \(x\)-axis for \(a \leq x \leq b\) is

\[
SA = 2\pi \int_a^b y \, ds
\]
Find the area of the surface formed by revolving the graph of $f(x) = x^2$ on the interval $[0, \sqrt{2}]$ about the $y$-axis.

Find the area of the surface formed by revolving the graph of $x = \frac{1}{2} y^2 + 2$ on the interval $[2, 6]$ about the $x$-axis.
Find the area of the surface formed by revolving the graph of
\[ f(x) = \sqrt{x} \]
on the interval \([4, 9]\) about the \(x\)-axis.