PROBLEM 12: Determine whether the following series converge or diverge

(i) \( \sum_{n=1}^{\infty} (\ln(2n) - \ln n) \)  
(ii) \( \sum_{n=1}^{\infty} \frac{1}{n^3} \)  
(iii) \( \sum_{n=1}^{\infty} \frac{(\cos(n))^2}{\sqrt{n^5}} \)

You must explain your reasoning for each series, even if you can deduce the answer by process of elimination.

(a) All series converge  
(b) (i) and (iii) diverge; (ii) converges  
(c) (i) and (ii) diverge; (iii) converges  
(d) All series diverge  
(e) (ii) and (iii) converge; (i) diverges  
(f) (ii) converges; (i) and (iii) diverge
9. Which of the assertions below hold for the following series:

\[ I : \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}} \quad II : \sum_{n=1}^{\infty} \frac{n}{\sqrt{7n^5 - 6n}} \quad III : \sum_{n=0}^{\infty} \frac{2^n - 5^n}{3^n + 4^n} \]

(a) I, II, III are convergent   (b) I, II, III are divergent   (c) only I converges
(d) only I and II converge   (e) only I and III diverge   (f) only III converges
2. Determine whether the following series are convergent or divergent. For full credit be sure to explain your reasoning and tell what test was used.

\[ \sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2} \quad \left( \sum_{n=1}^{\infty} \frac{8n}{5+7n} \right) \quad \left( \sum_{n=1}^{\infty} \frac{\arctan(n)}{\sqrt{n^7}} \right) \]

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<tr>
<th>I</th>
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<td>A</td>
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<td>H</td>
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</tbody>
</table>
14 Which of the following series converge?

(I) \( \sum_{n=2}^{\infty} \frac{\ln n}{n^3} \) \hspace{1cm} (II) \( \sum_{n=2}^{\infty} \frac{n^3}{\ln n} \) \hspace{1cm} (III) \( \sum_{n=1}^{\infty} \frac{n}{2^n} \) \hspace{1cm} (IV) \( \sum_{n=1}^{\infty} e^{1/n} \)

(A) I & II \hspace{1cm} (B) I & III \hspace{1cm} (C) I & IV \hspace{1cm} (D) II & III

(E) II & IV \hspace{1cm} (F) III & IV \hspace{1cm} (G) all four of them \hspace{1cm} (H) none of them
14. The series
\[
\sum_{n=1}^{\infty} \frac{10n^3 2^n}{n^4}
\]
(a) converges by the alternating series test.
(b) diverges by the alternating series test.
(c) converges by the ratio test.
(d) diverges by the ratio test.
(e) converges because the terms approach 0.
(f) diverges because the terms do not approach 0.
10. Which statement is true for

\[
\begin{align*}
(I) & \quad \sum_{n=1}^{\infty} e^n \\
(II) & \quad \sum_{n=1}^{\infty} \frac{1}{n^e} \\
(III) & \quad \sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} \\
(IV) & \quad \sum_{n=1}^{\infty} \frac{n^2}{n^2 + 1}.
\end{align*}
\]

(A) All four series converge.  \quad \quad \quad (B) None of the series converge.
(C) Only (I) and (II) converge.  \quad \quad \quad (D) Only (I) and (III) converge.
(E) Only (I) and (IV) converge.  \quad \quad \quad (F) Only (II) and (III) converge.
(G) Only (II) and (IV) converge.  \quad \quad \quad (H) Only (III) and (IV) converge.
16. The series $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

(a) converges because the terms approach 0.
(b) diverges because the terms do not approach 0.
(c) converges by the alternating series test.
(d) diverges by the alternating series test.
(e) converges by the comparison test.
(f) diverges by the geometric series test.
17. The series \( \sum_{n=1}^{\infty} (-1)^n \frac{(n + 3)2^n}{3^n + 100} \)

(a) converges absolutely by the ratio test.
(b) converges conditionally (but not absolutely) by the ratio test.
(c) diverges by the ratio test.
(d) converges absolutely by comparison with \( \sum_{n=1}^{\infty} \frac{1}{3^n} \).
(e) converges conditionally (but not absolutely) by comparison with \( \sum_{n=1}^{\infty} (-1)^n \frac{n+3}{3^n} \).
(f) diverges by comparison with \( \sum_{n=1}^{\infty} (-1)^n 4^n \).
Math 104 - Rimmer
Hand in Hw # 11

ANSWERS:

Spring 2013 # 12:  E
Fall 2012 # 9:  D
Spring 2012 # 2:  C
Fall 2011 # 14:  B
Spring 2011 # 14:  C
Fall 2010 # 10:  F
Spring 2010 # 16:  B
Spring 2010 # 17:  C