PROBLEM 7: What is the centroid of the region bounded by the curves $y = x^2$ and $y = 8 - x^2$?

Hint: draw a picture of this region as your first step.

(a) $(-2, 3)$ (b) $(2, 5)$ (c) $(-1, 4)$ (d) $(0, 4)$ (e) $(0, 3)$ (f) $(1, 4)$
11. Suppose that the region bounded by \( y = 4 \tan(x^2) \) and the \( x \)-axis for \( 0 \leq x \leq \frac{\sqrt{\pi}}{2} \) is a thin homogeneous density plate of area \( A \). Then the \( x \)-coordinate of the center of mass of the plate is:

(a) \( \frac{2}{A} \pi^2 \)  
(b) \( \frac{2}{A} \pi \)  
(c) \( \frac{1}{A} \ln 2 \)  
(d) \( \frac{3}{A} \sqrt{\pi} \)  
(e) 0  
(f) \( \frac{e \pi}{2} \)
12. What is the area of the surface obtained by rotating the part of the curve \( y = \sqrt{4-x^2} \) from \( x = 0 \) to \( x = 1 \) around the \( x \)-axis?

A) \( 4\pi \)  
B) \( 2\pi \)  
C) \( \pi \)  
D) \( \sqrt{2}\pi \)  
E) \( 3\pi \)  
F) \( 8\pi \)
2. Find the length of the arc of the curve defined by \( y = \frac{2}{3} \sqrt{x^3} \) for \( 0 \leq x \leq 3 \).

(A) \( \frac{\pi}{2} \)  (B) \( \frac{\pi}{4} \)  (C) 4  (D) 5 \ln 3  (E) \( \frac{14}{3} \)  (F) \( \frac{1}{4} \)  (G) \( \frac{e}{8} \)  (H) \( \frac{\ln 3}{2} \)
9. Find the arc length of the graph of \( y = \frac{x^3}{3} + \frac{1}{4x} \) between \( x = 1 \) and \( x = 2 \). [Note: It may be helpful to use identities like \((x^2 + \frac{1}{4x^2})^2 = x^4 + \frac{1}{2} + \frac{1}{16x^4}\).]

(a) 0  
(b) 59/24  
(c) \(\frac{8}{27}(10\sqrt{10} - 1)\)  
(d) \(\pi \ln(2)\)  
(e) \(\frac{3}{8} + \ln(2)\)  
(f) It is divergent.
10. Consider the graph of $y = \ln(\cos(x))$ between $x = 0$ and $x = 1$. Which of the following integrals corresponds to the surface area of the object obtained by rotating this graph about the $x$-axis?

(a) $\int_0^1 2\pi \sqrt{1 + \ln(\cos(x))^2} \, dx$
(b) $\int_0^1 2\pi \ln(\sin(x)) \sqrt{1 + \sec^2(x)} \, dx$

(c) $\int_0^1 2\pi \cos(x) \ln(\sin(x)) \, dx$
(d) $\int_0^1 2\pi \sec(x) \ln(\cos(x)) \, dx$

(e) $\int_0^1 2\pi x^2 \sin(x) \cos(x) \ln(x) \, dx$
(f) $\int_0^1 2\pi \sin^2(x) \sqrt{1 + \ln(x)^2} \, dx$
7. What is the arclength of the part of the curve $y = \frac{1}{12} e^x + 3e^{-x}$ for $\ln 2 \leq x \leq \ln 4$?

(A) $\frac{5}{12}$  (B) $\frac{1}{2}$  (C) $\frac{7}{12}$  (D) $\frac{2}{3}$  (E) $\frac{3}{4}$  (F) $\frac{5}{6}$  (G) $\frac{11}{12}$  (H) 1
10. An artist is designing a wine glass in a flower shape, which can be generated by rotating the region bounded by $y = \sqrt{x}$ and $x = y$, between $x = 0$ and $x = 1$, about $x$-axis. What is the surface area (which contains both the inside and the outside surfaces) of such a glass?

(a) $\left(\frac{8\sqrt{2} - 4}{3} + \sqrt{2}\right)\pi$
(b) $\left(\frac{8\sqrt{2} - 4}{3} + \sqrt{5}\right)\pi$
(c) $\left(\frac{8\sqrt{2} - 4}{3} + 1\right)\pi$
(d) $\left(\frac{5\sqrt{5} - 1}{6} + \sqrt{2}\right)\pi$
(e) $\left(\frac{5\sqrt{5} - 1}{6} + \sqrt{5}\right)\pi$
(f) $\left(\frac{5\sqrt{5} - 1}{6} + 1\right)\pi$
2. Find the volume of the solid obtained by rotating the region bounded by the curves 
\[ y = e^{x^2} \quad \text{and} \quad y = 0 \quad \text{and} \quad x = 0 \quad \text{and} \quad x = 2 \]
about the \( y \)-axis.
A.) \( 4\pi e^4 \) B.) \( 2\pi e^4 \) C.) \( 2\pi (e^4 - 1) \) D.) \( \pi (e^4 - 1) \) E.) \( \pi \sqrt{e} \) F.) \( \pi e \)
1. Find the volume of the solid obtained by rotating the region bounded by the curves 

\[ y = x^2, \quad y = 0, \quad x = 2 \]

about the line \( x = 4 \).

A.) \( 10\pi/3 \) \quad B.) \( 16\pi/3 \) \quad C.) \( 20\pi/3 \) \quad D.) \( 32\pi/3 \) \quad E.) \( 40\pi/3 \) \quad F.) \( 64\pi/3 \)
ANSWERS:

Spring 2013 # 7: D
Fall 2012 # 11: C
SPRING 2012 # 12: A
FALL 2011 # 2: E
SPRING 2011 # 9: B
SPRING 2011 # 10: D
FALL 2010 # 7: G
SPRING 2010 # 10: D
SPRING 2007 # 2: D
SPRING 2006 # 1: E