**Alternating Series Estimation Theorem**

If the alternating series \( \sum_{n=1}^{\infty} (-1)^{n-1} b_n \) (where \( b_n > 0 \)) satisfies:

1. \( \lim_{n \to \infty} b_n = 0 \)
2. \( \{b_n\} \) is a decreasing sequence

then \( |R_n| = |s - s_n| \leq b_{n+1} \)

The size of the error is at most ________________.

The actual sum is between ______________

The error has ________________.

The error committed in using the 9th partial sum to approximate the total sum is \( R_9 \)

The size of this error is at most the size of the first omitted term.

\[
|R_9| = |s - s_9| \leq \frac{1}{100} \Rightarrow -\frac{1}{100} \leq s - s_9 \leq \frac{1}{100}
\]

\[
s_9 - \frac{1}{100} \leq s \leq s_9 + \frac{1}{100}
\]

The actual sum is between \( s_n - b_{n+1} \) and \( s_n + b_{n+1} \).

The sign of the error is the sign of the first omitted term.

\[ R_9 = s - s_9 < 0 \quad \Rightarrow \quad s_9 > s \quad s_9 \text{ is an overestimate} \]

\[
\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n^2} = 1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25} - \frac{1}{36} + \frac{1}{49} - \frac{1}{64} + \frac{1}{81} - \frac{1}{100} + \frac{1}{121} - \frac{1}{144} \cdots
\]

\[
\underbrace{s_9}_{R_9} \quad \underbrace{\ldots}_{s}
\]
9. Which of the following is the best approximation of \( \ln(\frac{101}{100}) \)?

(A) 0  (B) \( \frac{1}{10} \)  (C) \( \frac{5}{100} \)  (D) \( \frac{9}{100} \)  (E) \( \frac{95}{1000} \)  (F) \( \frac{99}{1000} \)  (G) \( \frac{999}{10000} \)  (H) \( \frac{155}{10000} \)

\[
\ln (1-x) = -\sum_{n=1}^{\infty} \frac{x^n}{n+1} = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \cdots \quad \text{with } R = 1
\]
The Remainder Estimation Theorem. If there is a positive constant $M$ such that $|f^{(n+1)}(t)| \leq M$ for all $t$ between $x$ and $a$, inclusive, then the remainder term $R_n(x)$ in Taylor’s Theorem satisfies the inequality

$$|R_n(x)| \leq M \frac{|x - a|^{n+1}}{(n+1)!}.$$

If this inequality holds for every $n$ and the other conditions of Taylor’s Theorem are satisfied by $f$, then the series converges to $f(x)$.

Consider the polynomial $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$ as an approximation to $e^x$ on the interval $-2 \leq x \leq 2$. What is the best bound on the error for this estimate that is given by Taylor’s inequality?

(a) $\frac{1}{24}$  (b) $\frac{e}{12}$  (c) $\frac{2e^2}{3}$  (d) $\frac{e^3}{4}$  (e) $\frac{3e^4}{2}$  (f) $e^5$