1. Calculate the average value of the function \( g(x, y) = 12xyz^2 \) over the box defined by \( \{(x, y, z) \mid 1 \leq x \leq 3, \ 0 \leq y \leq 1, \ -1 \leq z \leq 2\} \):

   (A) 0  \hspace{1cm} (B) 2  \hspace{1cm} (C) 3
   (D) 4  \hspace{1cm} (E) 12 \hspace{1cm} (F) 13

2. Water is being poured into a container shaped like the paraboloid \( z = x^2 + y^2 \) at a constant rate (where \( x, y \) and \( z \) are all measured in meters). How much longer will it take to fill the container to a height of 3 meters than to fill it to a height of 1 meter?

   (A) \( \sqrt{3} \) times as long \hspace{1cm} (B) 3 times as long \hspace{1cm} (C) \( 3\sqrt{3} \) times as long
   (D) 9 times as long \hspace{1cm} (E) \( 9\sqrt{3} \) times as long \hspace{1cm} (F) 27 times as long
3. Calculate the work done by the force field \( F = (5y, 7x^2) \) on a particle that moves along the curve given by \( x = t^2, \ y = 2t^3 \) for \( 0 \leq t \leq 1 \).

(A) 4  
(B) 5  
(C) 10  
(D) 15  
(E) 16  
(F) 32

4. Calculate \( \iiint_S \frac{1}{\sqrt{x^2 + y^2 + z^2}} \, dV \) where \( S \) is the spherical shell bounded by the two spheres \( x^2 + y^2 + z^2 = 1 \) and \( x^2 + y^2 + z^2 = 9 \).

(A) \( 8\pi \)  
(B) \( 12\pi \)  
(C) \( 16\pi \)  
(D) \( 24\pi \)  
(E) \( 30\pi \)  
(F) \( 36\pi \)
5. Calculate $\int_C (1 + \sin \pi y) \, dx + (2 + \pi x \cos \pi y) \, dy$ where the curve $C$ follows the ellipse $4x^2 + y^2 = 1$ from the point $(0, -1)$ to the point $(0, 1)$.

(A) 0  \hspace{1cm} (B) 1  \hspace{1cm} (C) 2  \\
(D) 4  \hspace{1cm} (E) 6  \hspace{1cm} (F) 8

6. Calculate $\int_C (y + e^{-2x}) \, dx + (2x - \cos^2 y) \, dy$ where $C$ is the circle of radius 2 centered at the point $(1, 3)$, traversed counterclockwise.

(A) $2\pi$  \hspace{1cm} (B) $4\pi$  \hspace{1cm} (C) $8\pi$  \\
(D) $2\pi e$  \hspace{1cm} (E) $4\pi e$  \hspace{1cm} (F) 0
7. Calculate $\int \int_R 4x^2 \, dA$ where $R$ is the region in the first quadrant bounded by the graphs of $y = 1/x$, $y = 4/x$, $x = y$ and $x = 9y$ (so $R$ is a region with four corners, at (1,1), (2,2), (6,2/3) and (3,1/3). Letting $u = xy$ and $v = x/y$ might help. Don’t forget...).

(A) 60  
(B) 120  
(C) 180  
(D) 225  
(E) 240  
(F) 1200

8. Let $H$ be the top half of the ball $x^2 + y^2 + z^2 \leq 4$ (i.e., the part where $z \geq 0$). Calculate $\int \int \int_H x^2 \, dV$

(A) $\frac{16\pi}{15}$  
(B) $\frac{64\pi}{15}$  
(C) $\frac{81\pi}{5}$  
(D) $\frac{81\pi}{15}$  
(E) $\frac{1250\pi}{3}$  
(F) $\frac{1250\pi}{15}$