7. Find the equation of the plane that is tangent to the surface
\[ \cos(y + x) - \sin(y + z) = \sin(z) - \cos(x) \]
at the point \((\pi, \pi, 0)\). What is the \(y\)-coordinate of the point where this tangent plane intersects the \(y\)-axis?

(A) 3  
(D) \(\sqrt{\pi}\)  
(G) \(\pi/\sqrt{2}\) 

(B) \(\pi\)  
(E) \(\sqrt{3}\)  
(F) 1 

(C) 0
5. The ellipsoid

\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \]

bounds a region of volume \( V = \frac{4}{3} \pi abc \). At a moment when \( a = 1 \), \( b = 3 \), and \( c = 5 \), \( a \) is changing at a rate of +2 and \( b \) is changing at a rate of −3, at what rate must \( c \) be changing for the volume to remain constant?

(A) −7  
(B) −5  
(C) −3  
(D) −1  
(E) 1  
(F) 3  
(G) 5  
(H) 7
Fall 2010

5. The function \( z = f(x, y) \) is given implicitly by the equation \( z^3 + z = x^2 + y^2 \). Note that when \( x = 1 \) and \( y = 1 \), \( z = 1 \) as well. Compute \( \frac{\partial f}{\partial x}(1,1) \).

A. \( \frac{-3}{2} \)  B. \(-1\)  C. \( \frac{-1}{2} \)  D. 0  E. \( \frac{1}{2} \)  F. 1  G. \( \frac{3}{2} \)  H. 2
6. Consider the surface \( z = x^2 + x + 2y^2 \). At what point \((x_0, y_0, z_0)\) is the tangent plane parallel to the plane \( x + 4y + z = 0 \). What is the \( z \) coordinate of that point?

Answer \( z_0 = \) 

A. \(-1\)  B. 0  C. 1  D. 2  E. 3  F. 4  G. 6  H. 7
7. Let \( f(x, y, z) = z(x - xy^2) \). At the point \((1, 1, 1)\), find the angle between the vector pointing in the direction of fastest increase of \( f(x, y, z) \) and the \( x\)-axis.

A. \(-1\)  B. \(-\frac{1}{2}\)  C. 0  D. 0  E. \frac{\pi}{6}\)  F. \frac{\pi}{4}\)  G. \frac{\pi}{3}\)  H. \frac{\pi}{2}\)
Fall 2009

6. Suppose \( z = f(x, y) \), where \( x = g(s, t), \ y = h(s, t) \). Suppose that we know that
\[
\begin{align*}
g(1, 2) &= 3, \quad g_s(1, 2) = -1, \quad g_t(1, 2) = 4, \\
h(1, 2) &= 6, \quad h_s(1, 2) = -5, \quad h_t(1, 2) = 10, \\
f_x(3, 6) &= 7, \quad f_y(3, 6) = 8.
\end{align*}
\]
Find \( \frac{\partial z}{\partial s} + \frac{\partial z}{\partial t} \) when \( s = 1 \) and \( t = 2 \).

(A) 60  (B) -60  (C) 61  
(D) -61  (E) 62  (F) -62
(7) Let \( f \) be the function

\[ f(x, y) = \ln(x + y) \]

for \((x, y) \in \mathbb{R}^2\) and \(x + y > 0\). A unit vector in \(\mathbb{R}^2\) is a vector \(u\) of length \(|u| = 1\). What is the maximum value of the directional derivative \(D_u(f)\) of \(f\) at the point \((x, y) = (2, -1)\) as \(u\) ranges over all unit vectors in \(\mathbb{R}^2\)?

(A) 1  
(B) \(1/2\)  
(C) \(\sqrt{2}\)  
(D) \(\sqrt{3}\)  
(E) \(\ln(2)\)  
(F) 0  
(G) none of the above
Spring 2008

19) Find the equation of the tangent plane to the surface

\[ 4x^4 + 2y^4 + z^4 = 22 \]

at the point \((1, 1, 2)\).

(A) \[ 4x + 2y + z = 8 \]

(B) \[ 2x + y + z = 5 \]

(C) \[ 2x + y + 4z = 11 \]

(D) \[ 2x + 2y + z = 7 \]

(E) \[ x + y + 4z = 10 \]

(F) \[ x + 2y + 4z = 11 \]

(G) none of the above
Fall 2008

(7) Let $T(x, y) = x^2 + y^2 - x - y$ be the temperature at the point $(x, y)$ in the plane. A lizard sitting at the point $(1, 3)$ wants to increase his surrounding temperature as quickly as possible. In which direction should he move?

(A) $(1, 1)$  (B) $(1, 3)$  (C) $(1, 5)$
(D) $(1, 7)$  (E) He should stay still.  (F) none of the above
Hand-In HW # 5 Answers:

Fall 2011 # 7: B

Spring 2011 # 5: B

Fall 2010 # 5: E

Fall 2010 # 6: D

Fall 2010 # 7: H

Fall 2009 # 6: C

Spring 2008 # 7: C

Spring 2008 # 19: C

Fall 2008 # 7: C